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EUROPE REVISITED *

BY

MERLE M. ODGERS [1]

Mr. President, members of The Franklin Institute, ladies and gentlemen: Tonight I bring with me to the Institute the greetings of an admiring neighbor, Girard College, a neighbor deeply grateful for the many rich opportunities offered to Philadelphia by your Institute and its renowned divisions.

Apart from the fact that I count it an honor and pleasure to address so distinguished a body as The Franklin Institute, this is a rare opportunity, seldom enjoyed by any but the professional travelogist, to talk about "my trips." It has been denied that travel broadens the mind, but nobody has ever denied that it lengthens the conversation and heightens the desire to talk.

When a person has been born and has worked his entire life in one city, it's good for him to travel. At least, that's a pleasant rationalization of what makes him say,

"... go I must, and if men ask you why,
You may put the blame on the stars and the sun and the white road and the sky!"

You may also throw a little blame on those who developed the automobile and petroleum industries. If your gasoline and your spirits are high and your car is doing its job, there's no travel more pleasant than what the motor car permits, whether your car takes you through the Jersey pines or the corners of Great Britain, "far removed from the whole world," as Virgil tells us in his Eclogues.

I had not been in Europe since just before the war. Once again it was my pleasure to have a steering wheel in my hands and, on two recent

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(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

trips with my family, somewhat enlarged since my pre-war trips, ride over the old Roman roads like the Via Aurelia and the more modern wonders like the Susten Pass. Kipling may be right in claiming that

> "Down to Gehenna or up to the throne,
> He travels the fastest who travels alone,"

but there's a lot of fun in seeing Europe with a wife and daughter and son.

The greatest early American traveller, who also became America's greatest diplomat, had no automobile, but he, too, started out from Philadelphia. The patron saint of this Institute made a greater impression on Europe by his presence there than any American has made before or since. He was, of course, on a mission. I had no mission, unlike many of my friends who carried brief cases and had their expenses paid.

I haven't the slightest doubt that Benjamin Franklin enjoyed himself in Europe. During the progress of the American Revolution (March 5, 1780) he wrote from France to George Washington: "Should peace arrive after another campaign or two, and afford us a little leisure, I should be happy to see your Excellency in Europe, and to accompany you, if my age and strength would permit, in visiting some of its ancient and most famous kingdoms. You would, on this side of the sea, enjoy the great reputation you have acquired, pure and free from those little shades that the jealousy and envy of a man's countrymen and contemporaries are ever endeavouring to cast over living merit. Here you would know, and enjoy, what posterity will say of Washington. For a thousand leagues have nearly the same effect with a thousand years." Yes, it's a pity that Franklin and Washington never had the chance to travel together.

Certainly the Father of his Country would not have shared the horror of Abigail Adams, the wife of John Adams, who in 1784 wrote to a friend about a dinner at Dr. Franklin's in Paris at which she met Madame Helvetius, the widow of the distinguished philosopher. "She entered the room," wrote Mrs. Adams, "with a careless, jaunty air; upon seeing ladies who were strangers to her, she bawled out, 'Ah! mon Dieu, where is Franklin? Why did you not tell me there were ladies here?' You must suppose her speaking all this in French. . . . She ran out of the room; when she returned, the Doctor entered at one door, she at the other; upon which she ran forward to him, caught him by the hand, 'Hélas! Franklin'; then gave him a double kiss, one upon each cheek, and another upon his forehead. When she went into the room to dine, she was placed between the Doctor and Mr. Adams. She carried on the chief of the conversation at dinner, frequently locking her hand into the Doctor's, and sometimes spreading her arms upon the backs of both the gentlemen's chairs, then throwing her arm carelessly upon the Doctor's neck. I should have been greatly astonished at this conduct, if the good

Doctor had not told me that in this lady I should see a genuine Frenchwoman, wholly free from affectation or stiffness of behavior, and one of the best women in the world. For this I must take the doctor's word; but I should have set her down for a very bad one, although sixty years of age, and a widow." But even Abigail thawed somewhat, for five months later she wrote to the same correspondent: "I have found my taste reconciling itself to habits, customs, and fashions, which at first disgusted me."

To be sure, Franklin had made earlier visits to Europe. He had gone to England at a time when Americans visiting the home country did not think of themselves as foreigners. As Philip Rahv would doubtless remind us, this was in marked contrast to later attitudes, for he cites Sherwood Anderson's recollection that when he was a boy in Ohio, going to Europe meant "something tremendous. . . . It was of infinitely more importance than, let us say, getting married. Such and such a one had been to Europe three times. He was consulted upon all occasions, and was allowed to sit on the platform at political meetings. . . ."

In our day Europeans are our neighbors. It is true that a New York customs man who knew I had been abroad in two successive summers asked me with a smile, "Are you anti-American?" The acquaintance of my family with all of our forty-eight states is evidence to the contrary, I think, but Europe no longer seems far away. The world is much smaller than when any of us were children, and our own nation is now part of a great Atlantic community that includes western Europe. Paris is as close as San Francisco and Rome no farther than Seattle, so far as our Atlantic community of interest is concerned.

But, even with our new attitude, the individual who enjoys traveling is likely to feel that he is throwing off the cares of the week's seven days and achieving a blessed separation from his telephone. He is likely to say, as Hazlitt did say, "I should . . . like well enough to spend the whole of my life in travelling abroad, if I could anywhere borrow another life to spend afterwards at home!" But a man's funds, even in American dollars, will determine just what traveling he may do.

Last year I took my own car, for a year earlier I had found that it was no longer the inexpensive thing to rent a car that it had been before the war. Our French Ford of 1948, which was not so small as the English Anglia, but not so large as our American Ford, was delightful up to the point when one tried to get baggage into it. It was a dream car when it came to negotiating mountain turns, for its wheel base was short. Its size was such that I could almost carry it upstairs to a hotel room, along with the suitcases. By contrast, our 1949 Chevrolet was sometimes excluded from European garages as too large.

Hotel prices, and prices in general, are higher than they were before the war. Unless the American lives and shops with his eyes, rather than his pocketbook, open, he finds his trip quite expensive. Inflation

in Europe might have attained even dizzier heights if it had not been for the Marshall Plan. Industrially, Europe looks busy and is busy. The Marshall Plan countries—that is, those making up the Organization for European Economic Cooperation with headquarters in Paris—have achieved with American help an industrial production quite beyond pre-war levels. Unfortunately, intra-European trade, at least until recently, has been almost a third below the pre-war level. If, in our program of European aid, we have felt at all pinched, we, nevertheless, realize that we have made sacrifices for our cousins across the sea which are small in contrast with the troubles that they have undergone and are undergoing. For them this is a great time of testing. It is a time of testing for us, too. Our consciences as much as our power, our friendliness as much as our size, and our acceptance of responsibilities to maintain decency, freedom, and stability as much as our fortunate resources will determine our future leadership and influence and position in a world still very much at odds with itself.

A first post-war day in London is something of a shock! Desolation everywhere! Windowless and roofless buildings from which all life has gone. It's still an interesting city—the complicated bus system, matins at St. Paul's, which can now be seen better because of the open bombed areas, street stands in Stepney, "car parks," as the English call them, where buildings used to stand before the blitz, queues for rationed articles, more regimentation in living and more uniforms, and yet, somehow more friendliness in the people, every evidence of their great wartime courage and grit, and wild flowers growing where only parts of buildings still stand, for everything returns to nature.

The Englishman is still fairly sure of himself, but he is becoming less so, for there is some bitterness, as well as austerity, in his life. He looks at his government and says, depending on his political views, either that what England needs is a change or that the cure for her troubles is more of what she now has. He is not quite the Englishman of an older day who could walk down a street anywhere in the world as though he owned it. Nowadays the American can walk down a street anywhere in the world as though he didn't care who owned it, and if he doesn't watch out, he may end up by owning it.

A good hotel in London with bread but no napkin at breakfast, and with napkin but no bread at dinner. In 1948, breakfast gloried in powdered eggs, their first for the rest of my family, the first for myself since 1918. True to form, the English cook made powdered eggs less palatable than they were thirty years ago, and I'm told that last year eggs were almost entirely off the menu.

When served, British meat is likely to approach paper thinness. Not even the Beef-eaters look as though they had been eating beef. I used to be amused by Nathaniel Hawthorne's ungallant description of the heavy-set English lady of fifty years. "She has," he assured us, "an

awful ponderosity of frame, not pulpy, like the looser development of our few fat women, but massive with solid beef and streaky tallow; so that (though struggling manfully against the idea) you inevitably think of her as made up of steaks and sirloins." Well, that isn't true now, for the lady's diet is all against it. There are still fat ladies, but they are not of "solid beef and streaky tallow." Their fat was born of carbohydrates. Good roast beef with Yorkshire pudding is almost as rare now as was once the American orange, which the late Mr. E. G. Budd told me he saw as a child only once a year at Christmas-time.

It is a different story across the Channel. France is only a few miles away; it could be a thousand. As far back as 1787, that veteran traveler, Arthur Young, wrote what is still true of France: "We have lived better in point of eating and drinking beyond a question. . . . The common cookery of the French gives great advantage. . . . The dessert at a French inn has no rival at an English one." That certainly is still true. I have considerable sympathy for the Britisher who doesn't like his diet and resents the possession of what seems to be more and better food by his recent enemies, the Germans. It is not easy to explain, even in over-simple terms, the food demoralization of Europe, which is almost as complete as the monetary demoralization. One is tempted, of course, to put much of it in demographic terms and say that there are twice as many Englishmen as there ought to be in the "tight little isle," but there are also twice as many Germans and twice as many Italians as ought to live in their home countries.

For the visitor, life in France is pleasant, very pleasant. Plenty of good food, with even the croissant back in 1949, the traditional wines and the wine card that is always a lesson in French geography, lunch on the Eiffel Tower, a carriage ride to the Invalides, the dancing bear in the zoo in the Bois de Boulogne (which some of our G. I.'s called the Boys of Bologny), the guignol show with excited little French children calling "Guignol, Guignol" whenever the robber appeared, the food stands in Rouen's Old Market Square where Jeanne d'Arc was burned, the children of the Chinese tourists eating prawns at Mt. St. Michel, a ride on the war-born amphibious boat at le Touquet Paris-Plage, bathing there and at Sables d'Or, St. Jean de Luz, and Le Lavandou, the story book picture town of old Carcassonne, the Place de la Carrière in Nancy, where tournaments were held of old, views of Mont Blanc and the glaciers of the French Alps, the great gorge through which one approaches the Grande Chartreuse, the little black puppy of three months near Vichy whose French master called him "whiskey" and whom my daughter renamed "young Overholt," the hall in the chateau at Chinon where Jeanne d'Arc pleaded with Charles VII, the donjon of the walled chateau at Domfront, which thrilled my son so much, the cathedral at Chartres seen as Henry Adams would have us see it, the bi-lingual Quaker meeting in Paris and an English sermon in Notre Dame given by the Archbishop

of Boston—both on the same Sunday morning, St. Guirec in Brittany and its seaside oratoire whose statue of the saint eligible young women have traditionally and irreverently stuck in the nose with pins for many years, the fascinatingly varied headwear of Brittany, a visit to Stephen Girard's parish of St. Remy in the northern end of Bordeaux, the Basque country and the rugged Pyrenees, the excursion to the sophisticated modern town of Pamplona in Spain, the old Roman wonders of the Provence, and the new wonder of the big Red Schoolhouse in Reims and its room where the Germans signed their unconditional surrender.

Of course, there are many grim reminders of war and how France suffered that Nazi tyranny might be beaten down: towns like St. Lo, virtually destroyed, Caen, William the Conqueror's place, in whose school Girard College boys have developed such an interest, St. Malo, partially ruined by the last-stand frenzy of a Nazi commander who was a demented dipsomaniac, war-time tanks and other rusting equipment occasionally encountered along the roads, the grim invasion beaches of the Channel, Dunkerque with its historic memories, and Mulhouse, near the German border, where my garage was one of only four buildings still standing in several blocks. I know Frenchmen who spent almost five years in German prisoner-of-war camps and others who served in various parts of the world in the Free French armies of De Gaulle. And I shall not forget with what pride the proprietor of a little hotel in St. Girons showed me a formal document from the American general in command of the area thanking him for his cooperation. His place was evidently a "stop" in a pre-liberation communications chain.

Benelux, made up of Belgiums, Netherlands, and Luxembourg, is a new concept for us, but it may be here to stay, and it already seems in miniature to exhibit the kind of customs union and economic union that we should like to see in western Europe as a whole. With many flags in evidence, there seemed to be much popular interest in Benelux. According to one Benelux spokesman, the distinctive features of its joint economy "are the logical result of certain natural phenomena, that is, coal as basic raw material, a favourable geographical position as regards trade and the finishing industries, and a dense population with good technical training," yet one official memorandum points out that "the difference in the economic structure of the three countries is one of the chief factors which will contribute to deriving the greatest possible benefit from joint co-operation." Of Benelux exports 14.5 per cent used to go to Germany, whereas two or three years after the war ended, this figure was still less than 3 per cent. Similarly, these three nations had previously received 18 per cent of their imports from Germany, and this was cut to less than a sixth in post-war days. When people in Benelux talk about a solution of the German problem and the reestablishing of normal traffic from and to the Ruhr Basin, they are talking about something that affects their daily lives.

Here in Benelux are memories of an older day. It is still possible to make the canals of Brugge provide a floating trip through past centuries that makes one feel as though he had been in a dream. Brussels reminds one of Edinburgh, for it just missed being a hub of civilization. Waterloo, a famous name, was simply the little place where Wellington slept after the battle, for the battle was actually fought at the nearby Mont St. Jean.

Miles away in Holland is that town of peace, the Hague—but why did we have to reach there on the eve of its 700th anniversary? That's the way things turn out when you're on the loose with a small car and a small pocketbook. Everything, including our hotel windows, was lit up with round glass oranges filled with electric bulbs. We had a grand time in town and at the nearby shore resort, Scheveningen. Like the lights honoring the House of Orange, some of the populace was lit up for a 700th anniversary.

But it was almost too much to find that, after the one day's drive from the Hague celebration to Volendam, we had come there on an important saint's day. When the Dutch hotel proprietor told me there were no keys for the rooms because the Nazis had taken all the keys, I felt again that all hotel guests were much the same the world over so far as taking keys was concerned. It was an interesting room we had, overlooking the Zuider Zee and a great expanse of lawn. There is no point in my trying to accustom myself to television, for, after two years, I am still getting used to bifocals; as a result, when I looked out on the lawn at Volendam the next morning and commented to my wife that I thought it was queer that a little white dog was barking so violently at a big brown rock on the lawn, she assured me that the mass on the lawn had just stretched after twelve hours of sleep and was a drunk who had been there since 7:30 the night before. Blest repose! I am sure that not all the celebrants did so well. Geography in Holland reveals different faiths and different holidays. Across the water, three miles away, we discovered that Marken, with its men of tight knickerbockers, had been quiet and going about its business. But on the mainland, Volendam was enjoying a holiday. Only a handful were in the church. The rest were devoutly honoring the saint by flirting, singing, lurching along the main dike or street arm in arm with companions, drinking one stiff Holland gin after another and dancing in the cafés. Nobody was self-conscious, so it was easy to note the old-fashioned wide trousers and narrow coats of the men and boys, and the ornate dresses and aprons and headpieces of the women, especially the square insets of flowered design which marked the dresses of the unmarried girls.

Of the Benelux countries Belgium seemed to be on top of the world in 1948, and less so in 1949. There seem to be more new American cars in Belgian hands than in all the rest of Europe. Of the cars with foreign licenses left by tourists in garages in Nancy, Rome, Rouen, or

Montreux, let us say, half of them carry Belgian license plates. Holland is not so well off physically. Luxembourg I know little about. In fact, I hardly knew I was there. From Malmedy and St. Vith I was seeking Bastogne, to pay my respects to the place where the American General Mac Auliffe had answered "Nuts" to the German demand for surrender. But before we discovered we were out of France, we were fifteen miles in Luxembourg. One would have thought that anything so Graustarkian would reveal itself sooner. In the Place de la Constitution in the capital itself there is something that ties together the two great wars. A monument, which had been erected in tribute to the 3000 Luxembourgers who served in the allied armies in World War I, had been destroyed by the Germans in 1940, but the town fathers have carefully reassembled what pieces were left of its base to serve as a memory lest they forget. This little two-by-four nation would, I am sure, not belittle the threat of Russia, but it can teach us that history can repeat itself—even with a third Teutonic showing.

Switzerland is still, of course, the number one vacationland. The mountains are what they always were, and the Swiss opened just three years ago through the Susten Pass what may easily be the finest mountain road in the world. After a decade away from Switzerland one revels in many a picture that reminds him of the best of the Canadian Rockies and our own Grand Tetons, in pedal boat rides on the lake at Luzerne, in the yodeling all-girl orchestras, in the horse-back riding and the echo at Interlaken, in Montreux and its lake and its Byronic Chateau de Chillon. Travellers in Europe know that Swiss prices have long been high, almost as high as those in this country. Yet Switzerland has long been impressively prosperous, and it still is.

And over the Alps from Switzerland is a great land. There St. Paul traveled, and St. Peter suffered martyrdom. Long before the war I loved Italy sincerely. Right now a number of things flash into my mind,—the Uffizi, the Pitti, the Palazzo Vecchio, and the rest that make Florence like nothing else anywhere. Charming Sorrento, looking across at Vesuvius, which seemed ominously quiet last summer. The student group at Perugia. The eager tourists at Assisi. Venice at sunset as one approaches in a vaporetto from the Lido. The breath-taking things in the Museo Nazionale at Naples. The House of Menander at Pompeii. Anacapri and Dr. Axel Munthe's San Michele. The best of all beefsteaks at Como and the best of all scaloppine somewhere in Rome. Frascati, Est Est Est, Falerno, and the other wines that old Horace either liked or should have liked. Thousands of bathing cabins on the beaches at Viareggio and Ostia. Rome with its priceless treasures old and new, some not so well known, like the American Academy on the Janiculum and the church of S. Clemente, itself virtually an archeological museum, run by the Irish Dominicans. The balconies at Ravello, far above the Mediterranean, especially at twilight, when the sound of

singing rises up the vine-covered hills from the little fishing boats as their lights go on. And Lake Garda and our favorite spot there, the little stretch of land that juts out into the lake and ends with Sirmione, where the Roman poet Catullus had a villa and wrote lines that are echoed in Tennyson's lines:

> "Row us out from Desenzano, to your Sirmione row! . . .
> Gazing at the Lydian laughter of the Garda Lake below
> Sweet Catullus' all-but-island, olive-silvery Sirmio!"

Italy offers surprises, of course. Streets and squares have been renamed in some places. It is difficult to get used to the ubiquitous Coca Cola and other cola drinks. That must be part of the heritage left by our G.I.'s. The mighty white monument in Rome which serves as the Tomb of the Unknown Soldier is still referred to by some Italians as "the wedding cake" because the G.I.'s called it that. The most delightfully pleasant surprise in all Italy comes to you when you see the famous bronze Baptistery doors in Florence, which were removed for safekeeping during the war. They are back again, these doors of Ghiberti that Michelangelo said were fit to be the doors of Heaven. And so they are! For you can now see the original gold that covered the bronze five centuries ago. It was rediscovered while the doors were off their hinges.

Like France, Italy looks fairly well. The shell holes in roads are gone, and war-destroyed bridges have been replaced, at least with temporary bridges. As in France, many of the ugly scars of war remain, but there is much new construction. Many homes, public buildings, commercial structures, and apartments are going up, sometimes with salvage stone. The people are working hard and with determination. Italy is recovering.

From many parts of Europe, including Italy, Europe's millions, our ancestors, came long ago to the New World. Over a century ago the American Ralph Waldo Emerson wrote in "English Traits" that he had told Thomas Carlyle that in America we were able to "play the game with immense advantage" because of "prodigious natural advantages . . . and that England, an old and exhausted island, must one day be contented like other parents to be strong only in her children." Well, I am sure that neither England nor Italy is contented "to be strong only in her children" who live here in America. And yet, see what these children have wrought. England without our help would be much worse off. Italy would not have attained its present level of recovery without America and the Marshall Plan—that is, the ERP plan (the European Recovery Program).

Italy is a good example of how the European Recovery Program has worked out. ERP supplies of wheat, flour, coal, petroleum, medicines, machinery, and raw materials of different types are sent from the

United States to Italy on an average of three ships per day. All this is done to help Italy to achieve economic stability. Ninety per cent of the supplies going to Italy are free, but Italian customers pay for them, and what they pay is deposited in the ERP Lire Fund in the Bank of Italy. The Italian government is not required to send this money to America, but it is at its disposal to spend upon industrial remodernization, land reclamation, home-building, railroad and maritime reconstruction, public works, and other heroic Italian efforts to create greater production, maximum employment, and a higher living standard.

It is costly to America, but it is not a relief hand-out, and the Italian people are responding as all of us had hoped. A visit to Italy shows that this post-war war against hopelessness, hunger, and chaos is being won, and we are encouraged in the hope that by 1952 the goal of economic stability will be attained in Italy, as in the other cooperating countries.

The picture should not be colored too rosily. The middle classes have been badly squeezed everywhere. Of the countries most visited by Americans, the United Kingdom seems most badly off. Optimistic Britishers will tell you: "Unemployment is barely noticeable; jobs are apparently secure; industry is finding it easy to earn profits; wages are relatively high; the necessities of life are more fairly distributed than ever before; and because they cannot buy many necessaries, many people have money which they can spend on things which they would otherwise regard almost as luxuries." An American traveler is not permitted to go uninstructed as to the significant increases in the numbers of British livestock, poultry, and pigs, in the amount of land under cultivation, and in the exports of engineering products and textiles. Harmfully restrictive labor union practices and employers' practices still plague England. Moreover, many a British meal looks like an austere triumph of mind over platter. I chuckled over the request of two Scottish girls who shared our small boat at the Blue Grotto in Capri in the Mediterranean that I make myself responsible for the tip for the entire group, but I was sobered almost immediately when they declined some food that we passed them, and one of them remarked that they had been eating so lightly at home that they no longer required much food. Yet Britishers look fit enough, as they would say.

No one who sees France and its people could believe, of course, that the recent war had treated her lightly, but we are likely to forget her tragic history in our own lifetime. The almost incredible poverty of fully employed persons that one can see in France has a history that is not to be explained in terms of political ineptness. Unlike some of her neighbors, victorious and otherwise, France "found herself first liable for repairing two-thirds of the damage which she had suffered in the 1914–18 war and next under the necessity of making preparations against the threat of war which was actually to break out in 1939, two successive burdens which absorbed a large part of her resources." Then came

more than four years of Nazi occupation with virtually no upkeep of French equipment and much loss through destruction and looting. It is little wonder that in its rehabilitation program the emphasis of France is on modernization of equipment. Even in agriculture this is vital, for the number of tractors in France for a given unit of cultivated land has been far below the number of tractors in some neighboring countries. It has always been pleasant to see the rich farm land of France as one drove its roads. Now, as one stops under a tree at the side of a road to eat his picnic lunch, he can think with pleasure of next year's bread as he looks on wheat fields and sees a piece of new farm equipment roll by. For industry and for life in general, power is still a problem, as any recent tourist could assure you. France, which was compelled to import a third of its power, had, before the war, in all forms of power the coal equivalent of only 2.1 tons per inhabitant, whereas England had 4.15 and the U. S. A., 5.1.

It has been a quaint notion that American touring has been a tremendous factor in the economic life of France, and that, as in some towns in Maine, Frenchmen lived most of the year on their take from summer visitors. Actually in the ten pre-war years, only a tenth of the foreign tourists in France were Americans, whereas four tenths were British. These last were almost entirely missing in 1948. Relaxed currency controls gave France a few more British tourists in 1949. In the old days, of course, the Americans in Paris, where most of them rambled hardly more than two miles from the Opera, had a better time than the Britishers in spite of, rather than because of, the fact that they outdid their British cousins in spending money. Nowadays, just as before the war, you can drive around some sections of France for three days or a week without seeing an American. But there is no denying the fact that tourism is a major source of foreign currency for France, and dollars are particularly attractive at this time.

At the customs shed in Cherbourg I met a New York lady who explained to me how she made her purchases. "When I go in a shop anywhere in Europe and have trouble getting what I want at my price, I just say, 'Marshall Plan!' I just yell at them, 'Marshall Plan!' I get what I want!" I don't know what moral I was supposed to draw from her story, but I am sure that hundreds of thousands of Europeans are truly appreciative of American post-war aid. The thin little British and French newspapers carry countless stories about the Economic Recovery Program. King George VI in his Christmas message spoke of American help, but from many humbler people have come touching tributes to America's sympathy for European difficulties. Few have been so gallant as something that happened to my little family in a small restaurant on the Left Bank in Paris. During our dinner a bottle of chilled champagne came to our table, and the waitress told me it came with the compliments of M. ——, who was dining with his wife a short

distance away. I went over to thank him. "Oh," he said, "I knew from your speech you were Americans, and I wanted you to have the champagne as a little sign of the great respect and gratitude I have for the United States, which has done so much for France during and after the Liberation."

Has the Marshall Plan been cheap? Far from it. Since the beginning of ERP, the Marshall Plan has meant the spending of more than ten billion dollars by Americans on European reconstruction. That means two weeks' pay for each American worker. Every hour the cost is many thousands of dollars. Rest assured that in all recorded history there is nothing like it. It is the great Christian Crusade of our time.

Most competent American observers in Europe praise our bipartisan foreign policy as something both remarkable and successful. American success in its post-war efforts to rehabilitate Europe, when viewed even by the least enthusiastic, may be considered gray, rather than black or white. Complete success would have been a miracle. But the absence of the effort might have meant chaos for Europe and tragedy for ourselves. Enlightened self-interest brought us the Marshall Plan. Its implementation was planned to have some terminus, of course. As ERP's Administrator, Paul G. Hoffman, once put it, ". . . our problem is not only to get Europe on its feet, but off our backs. Why should we be concerned about getting Europe back on its feet? . . . The answer, to quote a phrase from the Foreign Assistance Act itself, is that the then 'existing situation in Europe endangered lasting peace' . . ."

Many European nations, cursed though they have been with the spiral of inflation and with cruel consumer prices, have worked hard to get back on their feet. Some old evils have been struck down, and some new measures of international cooperation have been developed. Russia, to be sure, and the Communists of the world hate the Marshall Plan, but there is reason to hope that the new cooperation of western European nations and the success of their efforts will earn or force Russia's respect and will guarantee peace for longer than we dared hope two or three years ago.

Is the Marshall Plan worth what it is costing, assuming, of course, that in the time planned most of Europe will be back on its feet? A couple of post-war visits to Europe, preferably at least a year apart, would make it certain that your answer would be "yes." You would feel a new pride in being an American when you realized that to intelligent Europeans our Uncle Sam is not just Old Man Moneybags, but the great preserver of our civilization and everything it means.

Again and again I have been impressed by the unselfish and devoted service that men and women give to institutions which serve the public good. Busy men of great ability whom I know serve upon boards and committees of The Franklin Institute and similar societies, of hospitals, schools, colleges and universities, community chests, homes for the indi-

gent and aged, and a host of other worthwhile activities. These men feel intensely their trusteeship both of funds and of the causes themselves.

If we could only extend more widely this sense of trusteeship to our nation at large, there would be less cynicism about public affairs, and public affairs might be carried on with more general satisfaction. If the sense of trusteeship were carried to the world at large, America's present position in world affairs would be better understood, and we would not be placed in the position of helping Europe merely because we dislike Communism. After all, even enlightened self-interest can have a positive, rather than a negative, basis, and fear seems a trivial thing when compared with the Christian ethic.

Occasionally a man sees his own house, or even his whole neighborhood, from a new angle. So it is with the American who now revisits Europe and looks westward toward his homeland. He sees a rebirth in much of Europe because of the resilient spirit of Europe's peoples, and the sympathy and democratic idealism of his own America. He bows humbly when he thinks of the contribution of Europe to art, literature, music, and science. He shudders when he thinks how nearly European culture was to annihilation through some stupid tricks of fate. And he rejoices in the new dawn, and in what here in America is west of the dawn, for he knows that

> ". . . not by eastern windows only,
> When daylight comes, comes in the light,
> In front, the sun climbs slow, how slowly,
> But westward, look, the land is bright."

Foote Mineral Company Laboratories.—Expanded facilities for scientific research in the field of chemicals, ceramic materials, metals and alloys, electrode coatings and refined ores, were publicly displayed for the first time at an informal open house held at the new Foote Mineral Company Laboratories, on December 9th, 1949.

The new three story building on Lincoln Highway in Berwyn, near Paoli, Pa., has been equipped to carry out basic research, product development, and engineering studies related to improvement in equipment design and process control. The staff of 35 scientists and technicians is under the direction of Dr. S. C. Ogburn, Jr. Dr. Ogburn, as Manager of Research and Development, is in charge of all activities in this field carried out by the Company.

Several unique and highly specialized industrial development laboratories have been set up for the advancement of Foote products presently used by various industries. For ceramics, a section of the ground floor has been converted into a complete experimental unit in which all phases of theoretical and practical processing may be investigated, controlled, and determined. In this laboratory many significant projects in lithium chemistry are being carried on. Equipment is available for formulating, analyzing, testing, and producing a wide range of ceramic products. The welding laboratory is another complete unit, in which electrode coatings are prepared from selected raw materials, are applied to welding rods by extrusion and are then tested by welding with AC or DC welding machines. Also on the ground floor is a well equipped machine shop for the production of special machinery, a minerals separation laboratory, furnace room, and the crushing and grinding laboratory.

In addition to administrative offices, the metallurgical laboratory, engineering office and library are located on the first floor.

The top floor is devoted to chemical investigations, including analytical, inorganic, physical, and organic chemistry laboratories and glass blowing facilities. Further developmental work on lithium compounds, zirconium, and several other products is the subject of major projects.

This new and enlarged laboratory building is in line with the policy of Foote Mineral Company to expand its facilities to industry by providing basic research of far reaching scope for the development of new products and constant improvement of present products. To further this end, a separate building has been provided at the nearby Exton plant for pilot plant operations carried out under supervision of the Research and Development Department.

An edible covering on food bars in survival rations is being developed by the Army Quartermaster Corps for use in extremely cold climates, where heavy gloves or mittens make difficult the removal of metal-foil coverings. The new pectinate covering may be eaten with the bar itself. Bars include cereals of two types, date-apricot, sweetmeat, almond-chocolate, and sweet chocolate.

A new chemical method for removing salt-deposit scale from distilling units aboard ship, developed at the Naval Engineering Experiment Station at Annapolis, Md., has been adopted for Fleet use. The new method employs sodium acid sulfate, takes two hours and can be accomplished under way. Mechanical scale-removal methods now in use require stoppage of the fresh-water stills, take from four hours to two days to complete.

TRIGONOMETRIC SERIES APPLIED TO THE BENDING OF LONG RECTANGULAR PLATES TO A CYLINDRICAL SURFACE [1]

BY

FO-VAN CHANG [2]

ABSTRACT

In Professor Timoshenko's book, "Elastic Stability," [3] application of trigonometric series to the deflection of beams submitted to the simultaneous action of lateral loads and axial forces was discussed. However, the axial forces are independent of the lateral loads. In the book, "Theory of Plates and Shells" [4] by the same author, we have a slightly different case, namely the bending of long rectangular plates to a cylindrical surface. The supports are not free to approach. This certainly sets up axial forces dependent upon the lateral loads. How to solve such problems was fully discussed in Professor Timoshenko's book. This article is to present another method by using the trigonometric series.

1. INTRODUCTION

When a long rectangular plate is subjected to a transverse load that does not vary along the length of the plate, the deflection surface of a portion of such a plate at a considerable distance from the end can be assumed cylindrical, with the axis of the cylinder parallel to the length of the plate. We can therefore restrict ourselves to the investigation of the bending of an elemental strip cut from the plate by two planes perpendicular to the length of the plate and a unit distance apart. If the edges are free to approach each other as deflection occurs, the calculation of the deflection of the plate is essentially the same as the calculation of the deflection of beams. The only difference is that the flexural rigidity of a beam has to be replaced by another constant, the flexural rigidity of a plate. In practice the problem is much more complicated, since the plate is usually attached to the boundary and its edges are not free to move. Such kind of support sets up tensile reactions along the edges as soon as deflection occurs. These tensile reactions certainly depend upon the magnitude of deflection, or rather the lateral load. Our problem is thus reduced to the investigation of bending of an elemental strip submitted to the action of lateral load and of an axial force whose magnitude depends upon the lateral load. The method for handling such problems is given in Professor Timoshenko's book, "Theory of Plates and Shells." This article is to present a different method by using the trigonometric series.

[1] This article was written when the writer did his graduate work at Stanford University.

[2] Assistant Professor, Chekiang University, Hangchow, China.

[3] STEPHEN TIMOSHENKO, "Theory of Elastic Stability," New York, McGraw-Hill Book Co., 1936.

[4] STEPHEN TIMOSHENKO, "Theory of Plates and Shells," New York, McGraw-Hill Book Co., 1940.

2. CYLINDRICAL BENDING OF UNIFORMLY LOADED RECTANGULAR PLATES WITH SIMPLY SUPPORTED EDGES

Let Fig. 1 be the deflection curve of an elemental strip cut from the plate by two planes perpendicular to the length of the plate and a unit distance apart. Assume that the deflection curve can be represented by a sine series.

$$W = \sum_{n=1,3,5\cdots}^{\infty} a_n \sin \frac{n\pi x}{l}. \tag{1}$$

Each term, as can be seen easily, satisfies the end conditions since each term, together with its second derivative, becomes zero at the end of the strip. To determine the coefficients, a_1, a_3, $\cdots$ a_n, we use the principle

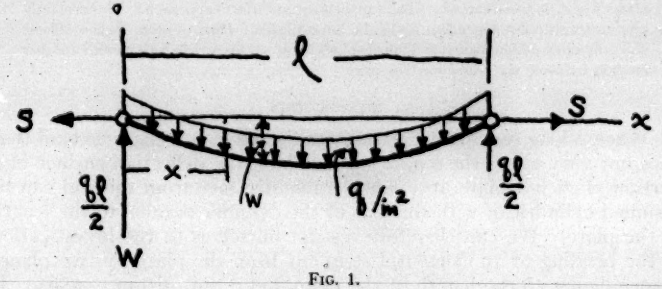

FIG. 1.

of virtual work. Let U_1 and U_2 be the strain energy stored in that strip due to axial force and bending respectively. The axis of the strip has its total elongation equal to:

$$\Delta L = \int_0^l (ds - dx) = \frac{1}{2}\int_0^l \left(\frac{dW}{dx}\right)^2 dx. \tag{2}$$

Differentiate expression 1.

$$\frac{dW}{dx} = \sum_{n=1,3,5\cdots}^{\infty} a_n \frac{n\pi}{l} \cos \frac{n\pi x}{l}. \tag{3}$$

Square 3 and substitute in 2.

$$\Delta L = \frac{\pi^2}{4l} \sum_{n=1,3,5\cdots}^{\infty} n^2 a_n^2.$$

Having known ΔL, we can determine S, the axial force.

$$\frac{S(1-\mu^2)l}{hE} = \frac{\pi^2}{4l} \sum_{n=1,3,5\cdots}^{\infty} n^2 a_n^2$$

Let h be the thickness of the plate, and

$\frac{E}{(1-\mu^2)}$ the modified modulus of elasticity for plate, then

$$U_1 = \tfrac{1}{2}S\Delta L = \frac{Eh\pi^4}{32(1-\mu^2)l^3}\left[\sum_{n=1,3,\cdots}^{\infty} n^2a_n^2\right]^2.$$

$$U_2 = \frac{1}{2}\frac{Eh^3}{12(1-\mu^2)}\int_0^l \left[\frac{d^2W}{dx^2}\right]^2 dx. \tag{4}$$

Differentiate expression 1 twice and substitute in Eq. 4.

$$U_2 = \frac{Eh^3\pi^4}{48(1-\mu^2)l^3}\sum_{n=1,3,5\cdots}^{\infty} n^4a_n^2.$$

Let the deflection curve undergo a very small displacement from its equilibrium position by making a change in $a_n \sin\frac{n\pi x}{l}$, say $(a_n+da_n)\sin\frac{n\pi x}{l}$, while other terms of the series are held unchanged. During this displacement, external forces do work equal to:

$$\int_0^l q\,dx(da_n)\sin\frac{n\pi x}{l} = \frac{2ql}{n\pi}da_n.$$

The changes of strain energy due to the increment da_n are:

$$\frac{\partial U_1}{\partial a_n}da_n = \frac{Eh\pi^4}{8(1-\mu^2)l^3}a_n n^2\left[\sum_{n=1,3,5\cdots}^{\infty} a_n^2n^2\right]da_n,$$

$$\frac{\partial U_2}{\partial a_n}da_n = \frac{Eh^3\pi^4}{24(1-\mu^2)l^3}a_n n^4 da_n.$$

By the principle of virtual work:

$$\frac{2ql}{n\pi}da_n = \frac{Eh^3\pi^4}{24(1-\mu^2)l^3}a_n n^4 da_n + \frac{Eh\pi^4}{8(1-\mu^2)l^3}a_n n^2\left[\sum_{n=1,3,5\cdots}^{\infty} a_n^2n^2\right]da_n.$$

Cancel da_n in each term.

$$\frac{2ql}{n\pi} = \frac{Eh^3\pi^4}{24(1-\mu^2)l^3}a_n n^4 + \frac{Eh\pi^4}{8(1-\mu^2)l^3}a_n n^2\sum_{n=1,3,5\cdots}^{\infty} a_n^2n^2.$$

To solve $a_1, a_3, \cdots a_n$, let $\sum_{n=1,3,5\cdots}^{\infty} a_n^2n^2 = S_n'$.

$$\frac{2ql}{n\pi} = \frac{Eh^3\pi^4}{24(1-\mu^2)l^3}a_n n^4 + \frac{Eh\pi^4}{8(1-\mu^2)l^3}a_n n^2 S_n'$$

$$\frac{2ql}{n\pi} = \frac{Eh\pi^4}{8(1-\mu^2)l^3}\left[\frac{h^2}{3}n^2 + S_n'\right]a_n n^2.$$

Since

$$S_n' = \sum_{n=1,3,5\cdots}^{\infty} \frac{[a_n n^2]^2}{n^2}$$

$$S_n' = \sum_{n=1,3,5\cdots}^{\infty} \left[\frac{16ql^4(1-\mu^2)}{nEh\pi^5} \cdot \frac{1}{\left(\frac{h^2}{3}n^2 + S_n'\right)}\right]^2 \frac{1}{n^2},$$

or

$$S_n' = \left[\frac{16ql^4(1-\mu^2)}{Eh\pi^5}\right]^2 \sum_{n=1,3\cdots}^{\infty} \frac{1}{\frac{h^4}{3^2}n^4\left[n^2 + \frac{3}{h^2}S_n'\right]^2}.$$

Let

$$\frac{3}{h^2}S_n' = S_n''$$

$$S_n'' = \left[\frac{48\sqrt{3}\,ql^4(1-\mu^2)}{Eh^4\pi^5}\right]^2 \sum_{n=1,3\cdots}^{\infty} \frac{1}{n^4[n^2 + S_n'']^2}. \tag{5}$$

Series 5, as can easily be seen, converges very rapidly, from which we have:

$$S_1'' = \left[\frac{48\sqrt{3}\,ql^4(1-\mu^2)}{Eh^4\pi^5}\right]^2 \frac{1}{(1+S_1'')^2} \tag{6}$$

$$S_3'' = \left[\frac{48\sqrt{3}\,ql^4(1-\mu^2)}{Eh^4\pi^5}\right]^2 \left[\frac{1}{(1+S_3'')^2} + \frac{1}{3^4(3^2+S_3'')^2}\right] \tag{7}$$

. . . . . .

$$S_n'' = \left[\frac{48\sqrt{3}\,ql^4(1-\mu^2)}{Eh^4\pi^5}\right]^2 \left[\frac{1}{(1+S_n'')^2} + \frac{1}{3^4(3^2+S_n'')^2} + \cdots + \frac{1}{n^4(n^2+S_n'')^2}\right]. \tag{8}$$

From Eqs. 6, 7 and 8, we can solve S_1'', S_3'', $\cdots$ S_n''. Having found S_1'', S_3'', $\cdots$ S_n'', we can get the coefficients of the trigonometric series by the following relations:

$$S_1'' = \frac{3}{h^2}a_1^2$$

$$S_3'' - S_1'' = \frac{3}{h^2}3^2 a_3^2$$

$$S_5'' - S_3'' = \frac{3}{h^2}5^2 a_5^2 \tag{9}$$

. . . . . .

$$S_n'' - S_{n-2}'' = \frac{3}{h^2}n^2 a_n^2.$$

As series 5 converges very rapidly, the trigonometric series observed from Eqs. 9 will also converge very rapidly. Consequently, by taking only the first few terms, we shall have a very good result for the deflection curve of the elemental strip cut from the long rectangular plate bent into a cylindrical surface.

As a numerical example, let us take the same example as given in Professor Timoshenko's book, "Theory of Plates and Shells," page 8.

A long rectangular steel plate 50 in. wide and $\frac{1}{2}$ in. thick carries a uniformly distributed load q, 20 psi. Find the deflection surface and the maximum stress. E is taken $30 \cdot 10^6$ psi.

Using Eq. 6,

$$S_1'' = \left[\frac{48\sqrt{3}\,20(50^4)(1-0.3^2)}{30(10^6)3.1416^5(\frac{1}{2})^4}\right]^2 \frac{1}{(1+S_1'')^2},$$

or

$$S_1'' = 271.633\,\frac{1}{(1+S_1'')^2}.$$

By trial,

$$S_1'' = 5.825.$$

Using Eq. 9, we get:

$$a_1 = 0.696 \text{ in.}$$

Using Eq. 7,

$$S_3'' = 271.663\left[\frac{1}{(1+S_3'')^2} + \frac{1}{3^4(3^2+S_3'')^2}\right].$$

$$S_3'' = 5.833.$$

Using Eq. 9, we get,

$$a_3 = 0.010 \text{ in.}$$

In the same manner we can take three terms for series 5 to obtain S_5''. But as the series converges very rapidly, even two terms will give us very good results.

Substituting a_1 and a_3 in Eq. 1, we get the deflection curve

$$W = 0.696 \sin\frac{\pi x}{l} + 0.010 \sin\frac{3\pi x}{l}.$$

The maximum deflection occurs at $x = \frac{1}{2}l$.

$$W_{\max} = 0.696 - 0.010 = 0.686 \text{ in.}$$

The result in Professor Timoshenko's book is 0.688 in. The direct tensile stress caused by the axial force S is

$$\sigma_1 = \frac{S}{h} = \frac{E\pi^2}{4(1-\mu^2)l^2} \sum_{n=1,3\cdots}^{\infty} a_n^2 n^2.$$

As

$$\sum_{n=1,3\cdots}^{\infty} a_n{}^2 n^2 = S_n{}'' \frac{h^2}{3},$$

$$\sigma_1 = \frac{30(10^6)3.1416^2(\frac{1}{2})^2}{4(50^2)(1-0.3^2)3}(5.833) = 15{,}814 \text{ psi.}$$

The result in the book is 15,830 psi.

Maximum bending stress:

$$\sigma_2 = \frac{6}{h^2} M_{\max} = \frac{6}{h^2}\left(\frac{d^2W}{dx^2}\right)_{x=l/2}\left(\frac{Eh^3}{12(1-\mu^2)}\right)$$

$$= \frac{Eh^3}{12(1-\mu^2)}\left(\frac{6}{h^2}\right)\left[0.696\,\frac{\pi^2}{l^2}\sin\frac{\pi x}{l} + 0.010\,\frac{3^2\pi^2}{l^2}\sin\frac{3\pi x}{l}\right]_{x=l/2}$$

$$= \frac{30(10^6)(\frac{1}{2})3.1416^2}{2(1-0.09)50^2}[0.696 - 3^2(0.010)] = 19{,}720 \text{ psi.}$$

The answer in Professor Timoshenko's book is 19,930 psi.

The maximum stress:

$$\sigma_{\max} = \sigma_1 + \sigma_2 = 15{,}814 + 19{,}720 = 35{,}534 \text{ psi.}$$

The maximum stress in the book is 35,760 psi. The difference is about 0.6 per cent.

CYLINDRICAL BENDING OF UNIFORMLY LOADED AND SIMPLY SUPPORTED RECTANGULAR PLATES ON ELASTIC FOUNDATION

Let Fig. 2 be the deflection curve of an elemental strip cut from the plate by two planes perpendicular to the length of the plate and a unit

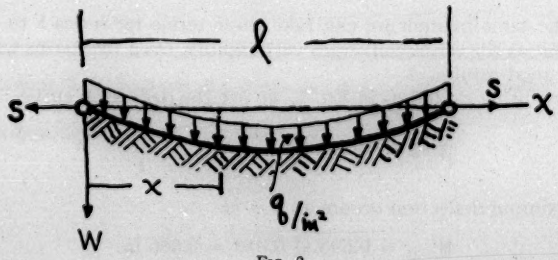

FIG. 2.

distance apart. Assume that the deflection curve can be represented by a sine series.

$$W = \sum_{n=1,3,5\cdots}^{\infty} a_n \sin\frac{n\pi x}{l}. \qquad (10)$$

Let U_1, U_2, and U_3 denote the strain energy stored in the system due to axial force, bending, and elastic foundation, respectively.

$$U_1 = \frac{Eh\pi^4}{32(1-\mu^2)l^3}\left[\sum_{n=1,3,5\cdots}^{\infty} n^2 a_n^2\right]^2$$

$$U_2 = \frac{Eh^3\pi^4}{48(1-\mu^2)l^3}\sum_{n=1,3,5\cdots}^{\infty} n^4 a_n^2$$

$$U_3 = \int_0^l \tfrac{1}{2}kW^2dx = \tfrac{1}{4}kl\sum_{n=1,3,5\cdots}^{\infty} a_n^2,$$

where k is the modulus of foundation. It represents the magnitude of reaction of the foundation per unit length of the elemental strip if the deflection is equal to unity.

By the principle of virtual work:

$$\frac{2ql}{n\pi} = \frac{Eh^3\pi^4}{24(1-\mu^2)l^3}a_n n^4 + \frac{Eh\pi^4}{8(1-\mu^2)l^3}a_n n^2\sum_{n=1,3\cdots}^{\infty} a_n^2 n^2 + \frac{kl}{2}a_n.$$

To solve $a_1, a_3, \cdots a_n$, let $S_n' = \sum_{n=1,3,5\cdots}^{\infty} a_n^2 n^2$.

$$S_n' = \left[\frac{16ql^4(1-\mu^2)}{Eh\pi^5}\right]^2 \sum_{n=1,3,5\cdots}^{\infty} \frac{1}{n^4\left[n^2\frac{h^2}{3} + S_n' + \frac{4k(1-\mu^2)l^4}{n^2Eh\pi^4}\right]^2}.$$

Put

$$S_n'' = \frac{3}{h^2}S_n'$$

$$K' = \frac{12k(1-\mu^2)l^4}{Eh^3\pi^4}$$

$$S_n'' = \left[\frac{48\sqrt{3}\,ql^4(1-\mu^2)}{Eh^4\pi^5}\right]^2 \sum_{n=1,3\cdots}^{\infty} \frac{1}{n^4\left[n^2 + S_n'' + \frac{K'}{n^2}\right]^2}. \tag{11}$$

Series 11 converges very rapidly from which we shall have:

$$S_1'' = \left[\frac{48\sqrt{3}\,l^4(1-\mu^2)}{Eh^4\pi^5}\right]^2 \frac{1}{[1 + S_1'' + K']^2} \tag{12}$$

$$S_3'' = \left[\frac{48\sqrt{3}\,l^4(1-\mu^2)}{Eh^4\pi^5}\right]^2 \left[\frac{1}{[1 + S_3'' + K']^2} + \frac{1}{3^4\left[3^2 + S_3'' + \frac{K'}{3^2}\right]^2}\right] \tag{13}$$

. . . . . .

$$S_n'' = \left[\frac{48\sqrt{3}\, l^4(1-\mu^2)}{Eh^4\pi^5}\right]^2 \left[\frac{1}{[1+S_n''+K']^2} + \frac{1}{3^4\left[3^2+S_n''+\frac{K'}{3^2}\right]^2} \cdots \frac{1}{n^4\left[n^2+S_n''+\frac{K'}{n^2}\right]^2}\right]. \quad (14)$$

Having Eqs. 12, 13 and 14, we can solve this involved problem much in the same way as we did for the case without the elastic foundation.

SOME ASPECTS OF THE FUNCTION, ORGANIZATION AND OPERATION OF INDUSTRIAL RESEARCH AND DEVELOPMENT LABORATORIES *

BY

DUDLEY E. CHAMBERS [1]

Gentlemen, your committee asked that I briefly discuss the following questions from the viewpoint of a large corporation: 1. how to decide when research is needed; 2. how to plan a program; 3. how to budget the program; and 4. what may be expected from the work.

In the course of my remarks I shall try to answer these questions, but, for reasons which I hope will become obvious, I shall tend to do so in a rather indirect manner.

For example, I should like to take time now to describe my concept of the several steps from exploratory or basic research through manufacture, because management action depends in part upon a sure appreciation of these steps.

In this concept,

Step 1 involves explorations for new physical facts without having applications in mind;

Step 2 involves the first applications of physical facts to produce new materials, new processes;

Step 3 includes the intensive developments that

(*a*) fit new materials, processes, or systems to specific fields of application

and

(*b*) modify and improve old materials, processes, or systems for specific fields of application;

Step 4 is the work concerned with specific product development;

Step 5 is concerned with specific product design; and

Step 6 is the operations involved in specific product manufacture and include

(*a*) the factory operations

and

(*b*) the test, inspection, and control operations.

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* Presented at the evening session of a joint meeting of the Philadelphia District Council of the American Society for Testing Materials and The Franklin Institute, on "Research for Management," held at The Franklin Institute, December 14, 1949.

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Perhaps an example would serve to illustrate somewhat more clearly the meaning of these steps.

Let us consider the progress through these steps of a new material. Suppose a gifted chemist who is interested in learning new facts about the mechanism of polymerization discovers something of significance. This fact may be the means which, properly applied, may allow a completely new material or series of materials to be created. The work of discovering the new fact, of producing for the first time the new materials and describing their general properties are what, for convenience in differentiating, I would define as "research." But it doesn't follow, necessarily, that the first bit of this new material is suitable for an application. It may have general properties which make it appear to have excellent promise for a certain field, but it is usually necessary to expend much more work to fit the new material for a specific field of application; indeed, work in this category, which I have called Step 3, may continue almost indefinitely to produce variations of the material for new applications as these crop up. It may be that one result of the work of Step 3 on this material produces an insulation of markedly superior properties. However, it will probably be found that while the material may be reasonably satisfactory to mould, it cannot be extruded or it cannot be worked in some other way without additional modification; I have called this modifying work Step 4—specific product development. When the new material has been evolved into, let us say, something really satisfactory for moulding operations, the product designer is free to apply it to his various problems, and I have called this work Step 5. Then, finally, comes Step 6, the manufacture of the specific product.

CONSIDERATIONS UNDERLYING TECHNICAL ORGANIZATION

Now few companies or groups desire or can afford to include all six of these steps in their work. Hence, broad decisions are first necessary to define the scope of technical operations; that is, which of, and the number of, the steps to be included in the company's work based on the available funds and upon the company's fields of interest, and to establish how the responsibility for each of the steps is to be divided among the various company groups.

Such decisions will vary greatly from company to company; there is no one best solution. But, in general, decisions concerning the organization must stem from considerations of the company's end products and the means by which it is believed the fields for these products can be most profitably exploited. To illustrate, I should like briefly to compare a telephone company, a steel company, and a large electrical manufacturing company.

Telephone companies tend to be very large and to deal with a wide range of technical subjects; yet their final interest is in providing communication—usually voice communication—and in making it possible

for a familiar looking instrument in one part of the world to enable one quickly and easily to communicate with whomever else one chooses in almost any other part of the world. For best results, such an undertaking requires technical endeavor in all (including the research) steps. As to organization, two factors generally influence the telephone company to go far towards setting up central technical facilities, establishing a relatively long-term technical plan, holding technical endeavor within this plan and expecting new technical developments to emanate from this close-knit central group. The first of these factors is the general necessity for producing standardized equipment in volume; the second is the relatively small influence any customer has in dictating the specifications of the equipment which lie between the final handsets.

With a steel company, however, it may be the case that despite a possible large dollar volume of business, the company has a minor interest in basic research. In few instances can steel companies afford to include all of the six steps in their work. You see, many steel companies are interested principally in the permanence and strength of their products under operating conditions, and when work is undertaken within such companies there must be a pretty good chance that the work will lead to results which will enhance these properties. But it is rarely possible to predict what the results of basic exploratory research will be. For example, it would be quite possible that work in solid state physics instituted with the hope of learning how to realize a greater fraction of the theoretical strengths of steels might, instead, yield knowledge which would be of aid in the field of magnetics. Hence, such a company rarely finds it possible to support such basic work.

A large electrical manufacturing company, in contrast to both of these other companies, sells thousands of products to many kinds of customers, and these products vary greatly as to type; for example: steam turbines, lamps, electronic equipment. Such a company must be sensitive to the wide range of needs of its many types of customers, and it must be willing to build one of a kind to order, or to mass produce a product. Its extremely wide range of product requires technical endeavor of a high order in many fields and new scientific facts or bits of technical information of a very wide variety usually find use, sooner or later, in such a company's products.

To emphasize this point, consider the metallurgical interests of the electrical company. The electrical company must be interested in the structural qualities of metals in normal atmospheres and at normal temperatures, and it must be interested in metals or other materials which will operate at the highest practicable operating temperatures, perhaps under conditions of severe erosion or corrosion (gas and steam turbines) or at very low operating temperatures. It is frequently interested in light weight. It must be interested in materials which will stand severe shock and prolonged conditions of vibration. It must be interested in

the magnetic properties (transient and permanent) of metals and in their conductivity (electrical and thermal). It must be interested in the behavior of metals used as electrical contacts and in materials used to maximimize or minimize electron emission. And with the possibility of power from atomic reactors, it must also be interested in the ability of materials to withstand prolonged conditions of bombardment by nuclear particles. The electrical company also has similar broad techncial interests in such fields as physics, chemistry, etc.

As a result of the various factors I have cited, our steel company strongly tends to include only Steps 3 through 6 in its work, while the telephone company and the electrical company, if they can afford it, tend to include all six of the steps in their work. And, I might interject parenthetically that, whereas a telephone company tends to include the first five steps in one "laboratory" organization and the sixth step in a "manufacturing" organization, an electrical company tends to center its basic research (the work of Steps 1 and 2) in a separate "research laboratory" and to give responsibility for Steps 3, 4, 5 and 6 to its several operating departments. These operating departments are organized to carry on a given kind of business, such as the lamp business, and, in addition to their sales and manufacturing groups have the engineering, engineering development and laboratory groups necessary successfully to carry out the technical work of Steps 3 through 6.

DIFFERENCES BETWEEN RESEARCH AND OPERATIONS

As one further bit of background, I should like to draw your attention to certain of the basic differences between "research" as defined by Steps 1 and 2 and "operations" as defined by Steps 3 through 6. In the case of the operations function, it is necessary to deal with some customer (a human). This customer has some kind of a system in which he is interested, and it is necessary to fit products into the system; hence, it is necessary to set up specifications to which a product must conform. Customers are normally interested in dates of delivery, and they certainly are interested in the lowest price; hence, an operating group must generally equal or better the competition in these respects. Thus, operations are highly organized endeavors; individuals in operating groups must be willing to accept assignments and must be willing to *compromise* factors to try to achieve a stipulated result. In general (although there are certainly exceptions to this) the rank and pay of an individual in an operating group tend to vary as the number of people he has working for him.

On the other hand, in the case of research one deals with nature and is concerned with specific truths as these are found. Since research is an exploratory endeavor, one cannot predict what will be found nor when it will be found; hence, one cannot set up dates on which one promises to deliver a research result; however, there is certainly a factor of urgency

in research because any one researcher (or company) realizes that others may be working in his field and that they may make the discoveries and may report before he does, if he is not diligent, and this can be a very strong *driving* force. Due to the character of exploratory research, it is necessary to pay what the work costs, whatever the figure may be (truth, being unique, cannot be compromised); however, research management has found that costs normally are lowest and results are best when it picks the very best men available and then backs these men in what they deem to be worth while doing. Exploratory research tends to be an individual endeavor in which an especially gifted scientist with pertinent training prospects for new facts, and it can hardly be expected that someone less informed can hope to "direct" his work or that he can hope to carry on much of his work through others. For these reasons and because exploratory research may be a relatively long-term endeavor, compensation of a scientist should be based on the worth and excellence of his contributions as judged over a fairly long period, relative to his opportunities.

Now, with this general background in mind, I should like to mention a very few of the basic points management must consider in connection with the various steps or areas of technical work.

Let us first assume that we have decided that our company is like the electrical manufacturing company I have mentioned; that is, its products are so technical and diverse in nature that it is likely that results from research of the type I have defined by Steps 1 and 2 would find use from time to time in its lines. Management must then arrange matters so this endeavor shall be as effective as possible, and, in this connection, the following comments may prove to be helpful.

As in other organized endeavors, the foremost factor in success is leadership. The laboratory director's leadership should come not only from requisite technical knowledge and business judgment, ability to recognize accomplishments and understand individual problems, but also from a devotion to scientific truth and an infectious enthusiasm which transmits itself to others. The possession by the director of these qualities will foster the growth of an esprit-de-corps in the laboratory which will lead to success if success is at all possible.

Of almost equal importance with leadership, and of much more significance than organization plans, facilities, accounting methods, etc., is the quality of the research staff people. The right people will make progress despite poor facilities or form of organization, but the wrong people will always turn in a poor job. Even more in exploratory research than in other endeavors, it requires the best ability to achieve results that are new and are a significant step ahead. Hence, the laboratory director is constantly looking first for persons of outstanding natural attributes and only secondly does he look at the scientific tools derived from training and experience which the man has collected. Of course,

the laboratory must achieve a balance of effort in the various fields such as physics, chemistry, metallurgy, etc., as is deemed to be required by the company's fields of interest, but it has been demonstrated many times that a truly outstanding man with good basic scientific training can contribute to a surprising degree in several fields. We have found that a true research man has outstanding scientific curiosity and is primarily interested in the unknown. Obstacles challenge him and arbitrary procedures and rules tend to irritate him. He seldom has the patience to carry a new idea through to a finished design. The exceptional is an inspiration to him and when he finds it he seeks to develop its peculiarities to the fullest extent. His imagination is of the philosophical type which enables him to coordinate his observations into hypotheses for experimental verification or disproof. He is always an optimist, for it is easy to think of many reasons why a proposed experiment would be futile; the man who makes the discoveries is the man who goes ahead and tries it; optimism is to research what vitamins are to the animal organism.

The third vital factor controlling the success of a laboratory is the wise choice of problems. In a business such as the electrical manufacturing business, there are always many, many times more problems in view than can be attacked, and it is vital that not only the *need* for work but the relative scientific *opportunity* for achieving results be evaluated by the best qualified personnel. There has been a need for a long time for a gravity eliminator and, one might say, the need is pressing. However, the scientist's opinion that the opportunity for producing results in this case is very small should be sufficient answer to some business men who might feel that there would be a large market for such a device. It is certainly essential that the company's long-term problems be continuously paraded before the director and his research staff, but for research of the type I have defined, the choice as to which of these problems should be attacked should be left to the laboratory director. And actually, he in turn will find it best simply to encourage the exceptional research men to work on the problems in their fields that interest them most and appeal to them as having the best opportunity for achieving results—the best chance for achieving the greatest pay-off. This is not to say that there are not problems of a research nature which require group organization and an acceptance by each member of the group of a somewhat specified portion of the problem; indeed, some of the efforts in the nuclear physics field are of such magnitude as to require such team play by brilliant individualists who normally would not care to so restrict their field of action but who have come to realize that the furthering of work in the field requires such organization. Of course, if the director is to have the best information concerning his company's plans and outstanding commercial, technical and manufacturing needs and is to develop the requisite perspective, it is of major aid if he is an officer

of the company and is a member of its advisory committee, for this insures that he will understand and have a hand in shaping company policies and it greatly aids in giving to top management a clearer understanding of his aims and methods.

Another vital factor in the success of a laboratory is the teamwork concept. While most exploratory research endeavors tend to be individual in nature, it is essential that there be a flow of information to and from each man for the purpose of providing him with pertinent help from others, and of insuring that his ideas and results are available to others when these can help. In some of the early German laboratories and indeed in some of the laboratories of this country, secrecy and what might be termed the "compartmental plan" prevailed. Each man worked behind locked doors and only the director knew what each was doing. Such a system forfeits the greatest advantage a large research laboratory, integrated into the structure of an industrial company, should have over scattered smaller groups which may or may not be affiliated with the company—the cumulative factor of teamwork. If each man's effort bears a certain relationship to the whole and if the knowledge resulting from his efforts is freely available to all others, the total output should rise not in arithmetical proportion to the number of workers but in some geometrical ratio. This flow of information should occur within the research laboratory and in a pertinent way to the interested personnel throughout the company.

In the organization of the laboratory, account must be taken of several facts. First, able creative research personnel are very rare. Second, experimental data and conclusions from these are usually obtained comparatively quickly; it is the process of obtaining the components and making the set-up that is usually so time consuming. Third, inspection of the annual budget of the usual research laboratory will reveal that the charge for facilities is relatively small. Therefore, all reasonable steps should be taken to utilize creative research ability to the greatest degree. Facilities and supporting personnel should be deliberately employed wherever possible to conserve the researcher's time and energy. Purchasing should be conducted and the laboratory shops should be operated so that the requirements of the researcher are promptly satisfied. There is far more to be gained ultimately from *quickly* producing a usable research result than there is in trying to economize in the supporting operations of the laboratory.

It is well if a recognized policy exists for liquidating the costs of a given research. Within a company, there is little to be gained and much may be lost by quibbling over amounts that have already been spent. Many companies consider it to be sound practice for the president to decide the approximate annual amount, on a relatively long-term base, that the company can afford to spend for research. This amount then is assessed against the operating groups, appropriately, and is made

available to the research laboratory for use in its work as it sees fit. And it is generally recognized that it is meaningless to require a detailed budget since the attractiveness of projects (the going assay of their *opportunities*) may change rapidly and radically with time and attractive new projects will undoubtedly appear on short notice as ideas occur to the researchers.

Despite the provisions for free flow of information between the research laboratory and the other company groups, it is necessary to establish definite responsibility for action to initiate transitions. In companies where the director of "research" is also chief engineer, the matter is simple. The laboratory findings may immediately be translated into technical instructions, by his office, which are binding on the factory. But when the operating divisions have the vertical form of organization and are each held responsible for their performance in their own field it is necessary to attach to the research laboratory's staff a group which acts as technical "salesmen" or liaison representatives. It should be the responsibility of this group to be familiar with the company's organization, facilities, principal personnel, principal problems and their status, on the one hand, and with the research organization, personnel, projects and their status on the other. This group should act to maintain regular contacts with the operating groups and to supplement those which other members of the laboratory staff may have. It should be its responsibility to initiate recommendations that projects be taken up by another division of the company and be terminated in the research laboratory. It should be its responsibility to follow a matter until successful transition is accomplished.

Well, so much for the "research laboratory"; now let us turn to the development, design and control functions.

As I have already pointed out, in the case of the electrical manufacturing company these functions are in the operations groups and thus are geared more or less closely with the current or relatively short-term needs of each of such groups. Since work in such groups must be so well integrated, since the individual must be willing to accept and to carry out assignments, leadership, again, is the outstanding requirement for success, but it must be leadership of a kind which utilizes many more of the instincts and tools of business management.

It is vitally important here, also, that we understand what traits to look for in our personnel. The successful design engineer likes to deal with certainties, to utilize materials whose properties he knows and has confidence in and to produce from them concrete results in new structures, new devices. He likes quickly to see the tangible results of his work. For economic reasons he must be a devotee of standardization. Optimism and imagination are certainly of great value to him, but they must be sternly ruled by sound judgment. Men for the advanced development work of Step 3 are hybrids, with various combinations of charac-

teristics which place them somewhere between the researcher and the designer.

In the case of our electrical manufacturing company we find that there is never any question as to whether technical work is required in Steps 3 through 6. The growth of the industry over the sixty to seventy years of its existence has served to make it very clear to the companies in the field that the public quickly rewards those who produce a better product, or those who can reduce the price of a product or those who can create a new product or new service for the public that it finds desirable. And since technical endeavor is so essential in this field to produce such results, our electrical company finds it necessary to conduct such work to protect and to expand its business.

The amount—the allowable cost—of technical work to be carried on in these steps varies widely among products, but it depends essentially upon the increased marketability of the products or upon the estimate of marketability of a new product as a result of the work. Whatever are the costs of the work of Steps 3 through 6, they should be charged as specifically as possible to the product which benefits from the work.

As for the establishment of the work program, this is best done through the close cooperation of the management, sales, technical and manufacturing groups. The decisions made must reflect management's judgment, based on the various bits of information from these sources, of the resulting commercial possibilities.

Whatever technical work in Steps 3 through 6 is decided upon, however, it will be found that, as in the case of the research laboratory, the personnel who have presumably been so carefully recruited should be backed up by supporting facilities and personnel to the end that results can be most quickly obtained. For, generally, the first to market reaps the largest profit.

In conclusion, I suggest that relatively few, even of the large industrial companies, can logically support directly much, if any, "research" of the type I have defined as Steps 1 and 2. If it is deemed that such research should be carried on, it must be recognized that research will rarely yield immediate financial returns; it is necessary to judge results over a relatively long period of time, and it must be expected that, if real research work is being carried on, the number of experimental failures will probably substantially outnumber the successes. On the other hand, the amount of technical work supported either inside or outside of company organizations has grown at an accelerating rate, especially over the past decade or so, as management has come more and more to realize the effectiveness of this kind of endeavor in protecting and expanding its businesses—and as the volume has grown, many companies have found it to be desirable to support work earlier and earlier on this technical scale; until, at present, many companies support work in Step 3.

There are major advantages to be gained by the public and by the company if the company can support work in all six steps. That is, historically, exploratory research has yielded, from time to time, outstanding technical advances. If such results come directly out of the work of the company research group, the close liaison between the research and other company groups will result in the quickest transition from research result to product reality. If such results do not come directly out of the work of the company research group but, instead, come from research carried on outside the company, the company research group is still in the best position to judge the possible implications of such work on the field of interest of the company so that suitable action can be taken; again, the final result is the quickest transition from research result to reality.

Now, in closing I will show you two general examples of results that can obtain from work carried out in all six of the technical steps. In these examples, taken from the experience of General Electric, you will see that certain results from the Research Laboratory played the primary roles in initiating the trains of events which have led to today's businesses, but you will also see that a *tremendous* amount of work in Steps 3, 4, 5 and 6 was necessary and was accomplished by the pertinent divisions of the operating groups of the company.

You know that General Electric from the date of its organization has been interested in the matter of light production. Early in the 1900's it was generally appreciated that tungsten, if it could be suitably worked, would be a much better filament material than carbon, and the company carried on a great deal of work in an effort to discover how this material could be used. The first tungsten filaments were made by mixing powdered tungsten with a binder to form a paste, squirting the paste through a die and then baking the filament to carbonize and to drive out the binder. The result was a pure but very fragile tungsten filament which required delicate and costly manipulation for its mounting and which yielded a lamp much more efficient but much more costly and fragile than the carbon filament lamp. It was obvious that the desirable way to form a tungsten filament was to get it into wire form and to draw it to size. But the problem was how to do it, for tungsten was a most brittle metal and had resisted all attempts to work it mechanically. In fact, the possibility of finding a solution to this problem was considered by many to be so improbable that the Lamp Department declined to finance the research. However, since the Research Laboratory is an independent department with certain funds allocated to it by the company's president, it was able to proceed with what it considered to be a promising lead. Meanwhile, the lamp development engineers went ahead with further developments related to the squirted filament. The Research Laboratory's work was long and costly, but many new facts

in the metallurgy of tungsten were discovered, and finally success was achieved. Then the Lamp Department gladly and quickly took over.

This work on ductile tungsten was carried out in 1911 by Dr. W. D. Coolidge and a few assistants. There was other work going on in the Research Laboratory at about this time, also, which found application in the lamp industry. For example, Dr. Irving Langmuir suggested that the life of tungsten lamps could be greatly improved and the efficiency increased by gas filling and concentrating the filament in coils, and that visible lamp darkening could be reduced by properly shaping the lamp bulb. These basic ideas, together with the ductile tungsten development, so increased the lamp's efficiency and sturdiness and opened up such possibilities for cost reduction that the Lamp Department was given competitive advantages over other forms of light production; these proved to be major factors in enabling the Lamp Department to grow rapidly and to reach its present substantial position in the lamp industry. Note that despite the vast technological development that has been necessary in the establishment of factories for lamp manufacture, the basic principles of today's tungsten lamp are those which came out of the Research Laboratory many years ago.

My last general example is the field of electronics. This is indeed a field that has depended upon work in all six of the technical steps, and I can mention only a *very* few of the significant results. You may remember that Edison, in his early work on the incandescent lamp, observed a phenomenon which has since been known as the "Edison Effect." He observed in some of his lamps a blue glow which rapidly disintegrated the filament near its terminals. He guessed that a current was passing through the space between the terminals, and proved this to be so by placing an electrode in the bulb and passing current between filament and electrode. He found that current would pass in one direction only. He also found that the effect appeared only in poorly exhausted lamps. This was as far as the state of knowledge permitted anyone to go at that time. Later, wireless telegraphy came, and Fleming, remembering Edison's lamp with the added electrode, and that it was a rectifier, tried it as a wireless detector and gave us the Fleming valve. Still later De Forest added a third electrode, the so-called grid, and changed the Fleming valve into an amplifier. All this time the tubes remained, like Edison's, poor vacuum devices, for as Edison found, they would not pass current if the vacuum was too high. In consequence they were also essentially low-voltage low-power tubes, since if more than about thirty volts were used, they would develop Edison's blue glow and destroy themselves. They were highly useful devices for radio reception, but that was all. J. J. Thompson had meanwhile discovered the electron, so it was clear that the Edison effect was the result of electron emission from a hot filament, but it was still a mystery as to why the presence of residual gas was necessary. It was generally thought that there was

some unknown chemical reaction between the hot filament and the gas which caused the electron emission. Richardson's work threw doubt on that theory, but it remained for Langmuir in our laboratory to give the complete explanation through his discovery of the space charge effect. He showed that electrons were freely emitted from a hot filament in the highest vacuum, but that the first to be emitted, being particles of negative electricity, charged negatively the space around the filament, and so, like the negative charge on the grid of a triode, limited the electron current to an amount so small that observers other than Richardson had failed to detect it. The effect of the gas in the poor-vacuum tubes was simply through its ionization to neutralize the space charge and so to permit much larger current to flow. Langmuir went on to show how, by bringing the anode close to the filament and using a high voltage, the electrons could be pulled out of the space so rapidly as to minimize the space charge effect and allow large currents to pass. Thus he gave us the power tube, capable of handling both high voltage and high current and therefore capable of amplifying high-frequency current to the amounts requisite for radio transmission.

After the discoveries of Doctors Langmuir, Thompson and Coolidge, which contributed so greatly to electron tubes, came the Hull magnetron, the further development of which by various groups in the world, including our Research Laboratory, made possible the radar, and radio and radar counter-measures eqiupments used so extensively in World War II. The thyratron and phanotron tube family invented by Dr. Hull opened up the industrial electronics field and today have extensive industrial applications. Electronics Park, at Syracuse, N. Y., the new home office and largest plant of the Electronics Department is one of the latest consequences of such research results.

Now, while my remarks may have emphasized to you the company benefits of integrated technical work, they failed to show, except by implication, the far larger benefits that accrue to the public. Consider the single example of the incandescent lamp. Suppose that tonight all illumination provided by incandescent tungsten lamps in the United States was instead provided from carbon filament lamps of the 1906 variety. Our yearly light bill would be increased by about 12 billions of dollars. Our fuel reserves would be depleted at the increased yearly equivalent coal rate of about 200 million tons. All of us would undoubtedly have much more trouble with our eyes than we do and automobiles might be much more lethal than they are.

THE OBLATE SPHEROIDAL WAVE FUNCTIONS[1]

BY

A. LEITNER[2] AND R. D. SPENCE[3]

1. INTRODUCTION

The oblate spheroidal wave functions arise as the eigenfunctions of such boundary value problems as the diffraction of sound and electromagnetic waves by circular disks and apertures, and the oscillations of spheroidal cavities. Recently no less than three papers, by Bouwkamp (1),[4] Spence (2) and Storruste and Wergeland (3), independently announced the solution of the problem of diffraction of a normally incident plane sound wave by disks and apertures, using identical methods. Extension of this solution and discussions of other problems based on the oblate spheroidal wave functions are under preparation by the present authors.

While a number of authors have discussed aspects of the functions (4–10), we feel that a relatively comprehensive analysis of these functions, which now command wide interest, is lacking. Thus Strutt (8) and Stratton (9), *et al*, give the expansions in terms of Legendre and spherical Bessel functions for the angular and radial functions, respectively, and the latter tabulate numerically some of the eigenvalues of the separation constant as well as some of the coefficients in these expansions; Bouwkamp (10) discusses the functions of order $m = 0$, presents a general method for computing the eigenvalues from the continued fractions and extends the tabulation of eigenvalues and coefficients.

In this paper the tabulation has been further extended up to eigenvalues for functions of order $m = 4$, and a large number of the angular *functions* themselves have been computed and plotted.

The viewpoint adopted here, in the analysis of the functions *at small arguments*, was previously used by Page. In his discussion of the prolate spheroidal functions (11), simple power series are used. This method is of advantage, in particular for the radial functions, since the series of spherical Bessel functions converge slowly at small arguments and do not explicitly show the properties of these functions near the singular points of the differential equations.

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[4] The boldface numbers in parentheses refer to the references appended to this paper.

While not all the material presented is original, it is hoped that it will serve to unify the subject by pointing out the interrelations among the approaches of the various authors in the field.

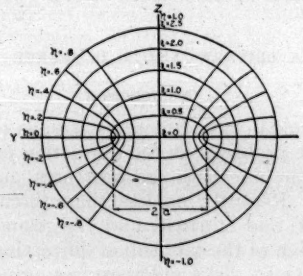


FIG. 1.

2. THE WAVE EQUATION IN OBLATE SPHEROIDAL COORDINATES

The oblate spheroidal coordinates η, ξ, φ, shown in Fig. 1, are related to the rectangular coordinates by the transformations

$$x = a[(1-\eta^2)(1+\xi^2)]^{\frac{1}{2}} \cos\varphi, \tag{1}$$

$$y = a[(1-\eta^2)(1+\xi^2)]^{\frac{1}{2}} \sin\varphi, \tag{2}$$

$$z = a\eta\xi, \tag{3}$$

where a is the radius of the focal circle, $0 \leqslant \varphi \leqslant 2\pi$, $-1 \leqslant \eta \leqslant 1$, and $0 \leqslant \xi < \infty$. In these coordinates the scalar wave equations, for wavelength λ,

$$\nabla^2\psi + k^2\psi = 0, \qquad k = \frac{2\pi}{\lambda}, \tag{4}$$

becomes

$$\frac{\partial}{\partial\eta}\left[(1-\eta^2)\frac{\partial\psi}{\partial\eta}\right] + \frac{\partial}{\partial\xi}\left[(1+\xi^2)\frac{\partial\psi}{\partial\xi}\right] + \left(\frac{1}{1-\eta^2} - \frac{1}{1+\xi^2}\right)\frac{\partial^2\psi}{\partial\varphi^2} + \epsilon^2(\eta^2+\xi^2)\psi = 0, \tag{5}$$

where

$$\epsilon = ka; \tag{6}$$

ψ may be expressed as the product

$$\psi_{lm} = u_{lm}(\eta)v_{lm}(\xi)\begin{matrix}\cos\\ \sin\end{matrix} m\varphi. \tag{7}$$

The functions $u_{lm}(\eta)$ and $v_{lm}(\xi)$ are the "angular" and "radial" spheroidal wave functions, respectively, and satisfy the ordinary differential

equations

$$\frac{d}{d\eta}\left[(1-\eta^2)\frac{du_{lm}}{d\eta}\right]-\left[\frac{m^2}{1-\eta^2}-\alpha_{lm}+c^2(1-\eta^2)\right]u_{lm}=0, \quad (8)$$

$$\frac{d}{d\xi}\left[(1+\xi^2)\frac{dv_{lm}}{d\xi}\right]+\left[\frac{m^2}{1+\xi^2}-\alpha_{lm}+c^2(1+\xi^2)\right]v_{lm}=0, \quad (9)$$

where α_{lm} is a separation constant whose eigenvalues are such that $u_{lm}(\eta)$ is finite at $\eta = \pm 1$.

Solutions of the vector wave equation

$$\nabla\nabla\cdot\mathbf{V}-\nabla\times\nabla\times\mathbf{V}+k^2\mathbf{V}=0 \quad (10)$$

may be constructed from the solutions of scalar wave equations. They are

$$\left.\begin{array}{lll}\mathbf{A}\psi, & \nabla\times\mathbf{A}\psi, & \nabla\times\nabla\times\mathbf{A}\psi,\\ \nabla\psi, & \nabla\times\mathbf{r}\psi, & \nabla\times\nabla\times\mathbf{r}\psi,\end{array}\right\} \quad (11)$$

where $\boldsymbol{A}$ and $\boldsymbol{r}$ are constant and radius vectors, respectively. The solutions involving the curl or curl curl are solenoidal while the other two are not.

3. THE FUNCTIONS $u_{lm}(\eta)$ AND $v_{lm}(\xi)$

Equations 8 and 9 may be obtained from the general equation

$$\frac{d}{dz}\left[(1-z^2)\frac{dF_{lm}}{dz}\right]-\left[\frac{m^2}{1-z^2}-\alpha_{lm}+c^2(1-z^2)\right]F_{lm}=0 \quad (12)$$

by replacing the complex variable z with η or $i\xi$, respectively. Equation 12 differs from the equation of the associated Legendre functions only in that it possesses an irregular singular point at $z = \infty$. The exponents associated with the regular singular points at $z = \pm 1$ are $\pm m/2$. A solution corresponding to the positive exponent will be called a function of the first kind; the other independent solution will be called a function of the second kind. Since the exponent difference is an integer the solutions of the second kind will possess logarithmic singularities at $z = \pm 1$. Since this point corresponds to $\eta = \pm 1$ or $\xi = \mp i$ it lies in the physical range of η but outside the physical range of ξ. Therefore, the angular functions are functions of the first kind while the radial functions are both the first and second kind.

3a. The Angular Functions

The analogy between (8) and the equation for the associated Legendre functions suggests series of the form

$$u_{lm}(\eta)=(1-\eta^2)^{m/2}\sum_{k=0}^{\infty}c_{2k}^{lm}(1-\eta^2)^k, \qquad (l-m)\text{ even}, \quad (13)$$

$$u_{lm}(\eta)=\eta(1-\eta^2)^{m/2}\sum_{k=0}^{\infty}c_{2k}^{lm}(1-\eta^2)^k, \qquad (l-m)\text{ odd}. \quad (14)$$

Also by analogy

$$\alpha_{lm} = l(l+1) + \gamma_{lm}, \tag{15}$$

where γ_{lm} is a function of ϵ^2. As in the case of the associated Legendre equation l is restricted to integral values equal to or greater than m. This notation follows that of Page (11). Stratton *et al.* (9) use the symbol l to denote $(l-m)$ as used here. The recursion formulas for the coefficients in (13) and (14) are

$$2k(2k+2m)c_{2k}^{lm} - [(2k-l+m-2)(2k+l+m-1) - \gamma_{lm}]c_{2k-2}^{lm} - \epsilon^2 c_{2k-4}^{lm} = 0, \qquad (l-m) \text{ even}, \tag{16}$$

$$2k(2k+2m)c_{2k}^{lm} - [(2k-l+m-1)(2k+l+m) - \gamma_{lm}]c_{2k-2}^{lm} - \epsilon^2 c_{2k-4}^{lm} = 0, \qquad (l-m) \text{ odd}, \tag{17}$$

with $c_0^{lm} \equiv 1$ in either case. The magnitude of the c_{2k}^{lm} begins to decrease when $2k > (l-m)+2$ if $(l-m)$ is even, and for $2k > (l-m)+1$ if $(l-m)$ is odd. For large values of k

$$c_{2k}^{lm}/c_{2k-2}^{lm} \to 1 + \epsilon^2/4k^2(c_{2k-2}^{lm}/c_{2k-4}^{lm}) \tag{18}$$

regardless of $(l-m)$. As k approaches infinity this ratio must either approach unity or approach zero as $-\epsilon^2/4k^2$. In the latter case the c_{2k}^{lm} alternate in sign at large k and the series (13) and (14) converge absolutely for all finite η. From (16) and (17) one may easily obtain the continued fractions

$$\gamma_{lm} = \cfrac{-l(l-m)\epsilon^2}{2(2l-1)+\gamma_{lm}+\cfrac{(l-2)(l-m-2)\epsilon^2}{4(2l-3)+\gamma_{lm}+\cfrac{(l-4)(l-m-4)\epsilon^2}{6(2l-5)+\gamma_{lm}+\cdots}}} + \cfrac{(l+2)(l-m+2)\epsilon^2}{2(2l+3)-\gamma_{lm}+\cfrac{(l+4)(l-m+4)\epsilon^2}{4(2l+5)-\gamma_{lm}+\cfrac{(l+6)(l-m+6)\epsilon^2}{6(2l+7)-\gamma_{lm}+\cdots}}}, \qquad (l-m) \text{ even}, \tag{19}$$

$$\gamma_{lm}=\cfrac{-(l-1)(l-m-1)\epsilon^2}{2(2l-1)+\gamma_{lm}+\cfrac{(l-3)(l-m-3)\epsilon^2}{4(2l-3)+\gamma_{lm}+\cfrac{(l-5)(l-m-5)\epsilon^2}{6(2l-5)+\gamma_{lm}+\cdot}}}$$

$$+\cfrac{(l+1)(l-m+1)\epsilon^2}{2(2l+3)-\gamma_{lm}+\cfrac{(l+3)(l-m+3)\epsilon^2}{2(2l+5)-\gamma_{lm}+\cfrac{(l+5)(l-m+5)\epsilon^2}{6(2l+7)-\gamma_{lm}+\cdot}}}$$

$(l-m)$ odd. (20)

In both (19) and (20) the first continued fraction contains only a finite number of terms while the second contains an infinite number of terms. Equations 19 and 20 constitute transcendental equations whose roots are the eigenvalues of γ_{lm}. The eigenvalues can be computed by cutting off the infinitely extending continued fraction and solving the resulting algebraic equation by successive approximations. By retaining more and more terms in the continued fraction one can obtain numerical values for γ_{lm} to any desired degree of accuracy.[5]

A series for the γ_{lm} in powers of ϵ^2 may be obtained (11) by substituting

$$\gamma_{lm} = \sum_{k=0}^{\infty} f_{2k}^{lm}\epsilon^{2k} \tag{21}$$

into (19) or (20) and collecting like powers of ϵ^2. One finds

$$f_2^{lm} = 2\frac{l(l+1)+m^2-1}{(2l-1)(2l+3)}, \tag{22}$$

$$f_4^{lm} = \frac{H(l-1)H(l)}{2(2l-1)} - \frac{H(l+1)H(l+2)}{2(2l+3)}, \tag{23}$$

$$f_6^{lm} = \frac{4m^2-1}{(2l-1)(2l+3)}\left[\frac{H(l-1)H(l)}{(2l-1)^2(2l-5)} - \frac{H(l+1)H(l+2)}{(2l+3)^2(2l+7)}\right], \tag{24}$$

[5] A method for rapidly finding the roots of such continued fractions was independently developed by Bouwkamp (10) and Blanch (12).

$$f_8{}^{lm} = \frac{H(l-1)H(l)}{2(2l-1)}\left[\frac{4(4m^2-1)^2}{(2l-5)^2(2l-1)^4(2l+3)^2} - \frac{f_4{}^{lm}}{2(2l-1)} + \frac{H(l-3)H(l-2)}{2\cdot 4(2l-1)(2l-3)}\right] - \frac{H(l+1)H(l+2)}{2(2l+3)} \times \left[\frac{4(4m^2-1)^2}{(2l-1)^2(2l+3)^4(2l+7)^2} + \frac{f_4{}^{lm}}{2(2l+3)} + \frac{H(l+3)H(l+4)}{2\cdot 4(2l+3)(2l+5)}\right], \quad (25)$$

where

$$H(x) = \frac{x^2-m^2}{4x^2-1}. \quad (26)$$

The series given in (21) does not appear to converge sufficiently rapidly to be useful if ϵ is greater than 5. For smaller values of ϵ it may be used to obtain a good approximation for γ_{lm} which may then be improved by use of (19) and (20).

The angular functions corresponding to a given ϵ are orthogonal in the interval $(-1, 1)$, that is

$$\int_{-1}^{1} u_{lm}(\eta)u_{l'm}(\eta)d\eta = \delta_{ll'}N_{lm}, \quad (27)$$

where

$$N_{lm} = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} c_{2i}^{lm}c_{2j}^{lm}\frac{2^{m+i+j+1}(m+i+j)!}{1\cdot 3\cdot 5\cdots[2(m+i+j)+1]}, \quad (l-m) \text{ even}, \quad (28)$$

$$N_{lm} = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} c_{2i}^{lm}c_{2j}^{lm}\frac{2^{m+i+j+1}(m+i+j)!}{1\cdot 3\cdot 5\cdots[2(m+i+j)+3]}, \quad (l-m) \text{ odd}. \quad (29)$$

3b. The Radial Functions

Both independent solutions of the radial equation (9) must be discussed, as noted above. We proceed by studying these functions first at small values and then at large values of the argument ξ. The radial functions of the first kind, ${}^{(1)}v_{lm}(\xi)$, are normalized such that

$${}^{(1)}v_{lm}(0) = 1, \quad (l-m) \text{ even}, \quad (30)$$

$${}^{(1)}v'_{lm}(0) = 1, \quad (l-m) \text{ odd}, \quad (31)$$

where the prime is used to denote differentiation with respect to ξ. This normalization is convenient in all boundary value problems involving the surface $\xi = 0$. The expansions

$${}^{(1)}v_{lm}(\xi) = \frac{(1+\xi^2)^{m/2}}{u_{lm}(0)}\sum_{k=0}^{\infty} c_{2k}^{lm}(1+\xi^2)^k, \quad (l-m) \text{ even}, \quad (32)$$

$${}^{(1)}v_{lm}(\xi) = \xi\frac{(1+\xi^2)^{m/2}}{u_{lm}'(0)}\sum_{k=0}^{\infty} c_{2k}^{lm}(1+\xi^2)^k, \quad (l-m) \text{ odd}, \quad (33)$$

are obtained by replacing η with $i\xi$ in (13) and (14) and dividing by the normalizing factors which makes the functions satisfy (30) and (31). An alternate expansion for ${}^{(1)}v_{lm}(\xi)$ is

$$ {}^{(1)}v_{lm}(\xi) = (1 + \xi^2)^{m/2} \sum_{n=0,1}{}' a_n^{lm} \xi^n. \tag{34} $$

The prime indicates summation over alternate n, starting with $n = 0$ if $(l - m)$ is even, with $n = 1$ if $(l - m)$ is odd. From (30) and (31) $a_0^{lm} = a_1^{lm} = 1$. The remaining coefficients are given by

$$ n(n-1)a_n^{lm} + [(n-2)(n+2m-1) + m(m+1) + \epsilon^2 - \alpha_{lm}]a_{n-2}^{lm} + \epsilon^2 a_{n-4}^{lm} = 0. \tag{35} $$

The determination of the radial functions of the second kind ${}^{(2)}v_{lm}(\xi)$ is considerably more complicated. One may proceed by use of the Wronskian of the radial functions, $W_{lm}(\xi)$. Its functional form follows directly from the radial differential Eq. 9

$$ W_{lm}(\xi) = W_{lm}(0)/(1 + \xi^2). \tag{36} $$

Its value at $\xi = 0$ cannot be determined as yet. It will be shown later that W_{lm} may be obtained from a study of the properties of the radial functions for large ξ. Since

$$ W_{lm}(\xi) = {}^{(1)}v_{lm}^2(\xi) \frac{d}{d\xi} \frac{{}^{(2)}v_{lm}(\xi)}{{}^{(1)}v_{lm}(\xi)}, \tag{37} $$

$$ {}^{(2)}v_{lm}(\xi) = W_{lm}(0) {}^{(1)}v_{lm}(\xi) \left[\int^{\xi} \frac{d\xi}{(1+\xi^2) {}^{(1)}v_{lm}^2(\xi)} + K_{lm} \right], \tag{38} $$

where K_{lm} is a constant of integration, as yet undetermined.

When $(l - m)$ is even it follows from (32)

$$ \frac{1}{(1+\xi^2) {}^{(1)}v_{lm}^2(\xi)} = \frac{u_{lm}^2(0)}{(1+\xi^2)^{m+1} \left[\sum_{k=0}^{\infty} c_{2k}^{lm} (1+\xi^2)^k \right]^2} $$

$$ = \frac{u_{lm}^2(0)}{(1+\xi^2)^{m+1}} \sum_{n=0}^{\infty} \frac{1}{n!} \left\{ \left[\frac{d}{d(1+\xi^2)} \right]^n \times \frac{1}{\left[\sum_{k=0}^{\infty} c_{2k}^{lm} (1+\xi^2)^k \right]^2} \right\}_{(1+\xi^2)=0} \cdot (1+\xi^2)^n \tag{39} $$

by Taylor expansion. The coefficients in the curly brackets are polynomials in the c_{2k}^{lm} and are easily found. For example, when

$$ \left.\begin{array}{lll} n = 0 & \text{it is} & 1 \\ n = 1 & \text{it is} & -2c_2^{lm} \\ n = 2 & \text{it is} & 6(c_2^{lm})^2 - 4c_4^{lm} \end{array}\right\} \tag{40} $$

When $(l - m)$ is odd one finds by use of (33)

$$\frac{1}{(1+\xi^2)^{(1)}v_{lm}^2(\xi)} = \frac{u_{lm}'^2(0)}{\xi^2(1+\xi^2)^{m+1}} \sum_{n=0}^{\infty} \frac{1}{n!} \left\{ \left[\frac{d}{d(1+\xi^2)} \right]^n \times \frac{1}{\left[\sum_{k=0}^{\infty} c_{2k}^{lm}(1+\xi^2)^k\right]^2} \right\}_{(1+\xi^2)=0} \cdot (1+\xi^2)^n. \tag{41}$$

The coefficients in the curly brackets are identical to those given for the even $(l - m)$.

The integral in (38) can now be evaluated. It has the general form

$$\beta_{lm} \tan^{-1}\xi + (1+\xi^2)^{-m}(\xi + \cdots), \qquad (l-m) \text{ even}, \tag{42}$$

$$\beta_{lm} \tan^{-1}\xi + (1+\xi^2)^{-m}(\xi^{-1} + \cdots), \qquad (l-m) \text{ odd}. \tag{43}$$

The dots in the parenthesis signify series in the ascending odd powers of ξ. The coefficients β_{lm} are completely determined, that is,

$$\beta_{lm} = u_{lm}^2(0) \sum_{n=0}^{m} \frac{(2m-2n)!}{n![2^{m-n}(m-n)!]^2} \times \left\{ \left[\frac{d}{d(1+\xi^2)} \right]^n \frac{1}{\left[\sum_{k=0}^{\infty} c_{2k}^{lm}(1+\xi^2)^k\right]^2} \right\}_{(1+\xi^2)=0}, \quad (l-m) \text{ even}, \tag{44}$$

$$\beta_{lm} = u_{lm}'^2(0) \sum_{n=0}^{m} \frac{(2m-2n+1)!}{n![2^{m-n}(m-n)!]^2} \times \left\{ \left[\frac{d}{d(1+\xi^2)} \right]^n \frac{1}{\left[\sum_{k=0}^{\infty} c_{2k}^{lm}(1+\xi^2)^k\right]^2} \right\}_{(1+\xi^2)=0}, \quad (l-m) \text{ odd}. \tag{45}$$

As a consequence, the form of the second kind radial function is known from (38) and one may write

$$^{(2)}v_{lm}(\xi) = Q_{lm}[^{(1)}v_{lm}(\xi)(\tan^{-1}\xi + C_{lm}) + g_{lm}(\xi)], \tag{46}$$

where

$$Q_{lm} = W_{lm}(0)\beta_{lm}, \qquad C_{lm} = K_{lm}\beta_{lm}^{-1}. \tag{47}$$

The values of C_{lm} as well as $W_{lm}(0)$ will arise from a discussion of the functions near the point at infinity which is given below.

The functions $g_{lm}(\xi)$ are odd in ξ when $(l - m)$ is even, and even when $(l - m)$ is odd, as follows clearly when (42) and (43) are substi-

tuted in the expression (38) for ${}^{(2)}v_{lm}(\xi)$. They have the form

$$g_{lm}(\xi) = (1 + \xi^2)^{-m/2} \sum_{n=1,0}^{\infty}{}' b_n^{lm} \xi^n, \tag{48}$$

where the summation is over odd n if $(l - m)$ is even and even n if $(l - m)$ is odd.

The $g_{lm}(\xi)$ satisfy the inhomogeneous radial equation

$$\frac{d}{d\xi}\left[(1 + \xi^2)\frac{dg_{lm}}{d\xi}\right] + \left[\frac{m^2}{1 + \xi^2} - \alpha_{lm} + \epsilon^2(1 + \xi^2)\right] g_{lm} = -2\frac{d}{d\xi}\,{}^{(1)}v_{lm} \tag{49}$$

from which a recursion formula may be found for the b_n^{lm}, namely

$$n(n-1)b_n^{lm} + [(n-2)(n+2m-1) + m(m-1) + \epsilon^2 - \alpha_{lm}]b_{n-2}^{lm} + \epsilon^2 b_{n-4}^{lm} = -2\left[\sum_{\lambda=0}^{m}\binom{m}{\lambda}(n-\lambda-1)a_{n-2\lambda-1}^{lm} + m\sum_{\lambda=0}^{m-1}\binom{m-1}{\lambda}a_{n-2\lambda-3}^{lm}\right]. \tag{50}$$

$\binom{m}{\lambda}$ denotes the binomial coefficients, the $a_{n-2\lambda-1}^{lm}$ are defined in (34) and (35), and $a_{n-2\lambda-1}^{lm} = 0$, when $(n - 2\lambda - 1)$ is negative. The initial coefficients b_0^{lm} and b_1^{lm} are as yet undetermined. However, since it follows from (46) and (48) that

$${}^{(2)}v_{lm}(0) = Q_{lm}C_{lm}, \quad {}^{(2)}v'_{lm}(0) = Q_{lm}(1 + b_1{}^{lm}), \quad (l-m) \text{ even}, \tag{51}$$

$${}^{(2)}v_{lm}(0) = Q_{lm}b_0^{lm}, \quad {}^{(2)}v'_{lm}(0) = Q_{lm}C_{lm}, \quad (l-m) \text{ odd}, \tag{52}$$

then it is clear from (30), (31) and the definition of the Wronskian that

$$W_{lm}(0) = Q_{lm}(1 + b_1^{lm}), \quad (l-m) \text{ even}, \tag{53}$$

$$W_{lm}(0) = -Q_{lm}b_0^{lm}, \quad (l-m) \text{ odd}, \tag{54}$$

consequently the coefficients b_1^{lm}, b_0^{lm} are determined by (47),

$$1 + b_1^{lm} = \beta_{lm}^{-1}, \quad (l-m) \text{ even}, \tag{55}$$

$$b_0^{lm} = -\beta_{lm}^{-1}, \quad (l-m) \text{ odd}, \tag{56}$$

and are independent of $W_{lm}(0)$.

Unlike the radial functions of the first kind, the radial functions of the second kind are in general neither even nor odd in ξ, cf. (51) and (52).

It is convenient to define in addition the radial functions of the third and fourth kinds as the linear combinations

$${}^{(3)}v_{lm}(\xi) = {}^{(1)}v_{lm}(\xi) + i\,{}^{(2)}v_{lm}(\xi), \tag{57}$$

$${}^{(4)}v_{lm}(\xi) = {}^{(1)}v_{lm}(\xi) - i\,{}^{(2)}v_{lm}(\xi). \tag{58}$$

These are required in the representation of diverging and converging waves.

The representation of he radial functions at large values of ξ is best accomplished by use of a definite integral due to Poole (5). He has shown that the integral

$$\int_a^b e^{i\epsilon\xi\eta}(1+\xi^2)^{m/2}(1-\eta^2)^{m/2}u_{lm}(\eta)d\eta \tag{59}$$

is a solution of the radial differential equation provided the limits are so chosen that

$$\left| e^{i\epsilon\xi\eta}(1-\eta^2)^{1+m/2}\left\{\left(ie\xi - \frac{m\eta}{1-\eta^2}\right)u_{lm}(\eta) - u'_{lm}(\eta)\right\}\right|_a^b = 0 \tag{60}$$

and ξ is such that the integral converges. The permitted limits for the contour are

$$\begin{aligned} a &= -1, & b &= 1, \\ a &= i\infty, & b &= 1, \\ a &= -1, & b &= i\infty. \end{aligned}$$

The integral (59) may be evaluated for the three different sets of limits by substituting the series (13) and (14) for $u_{lm}(\eta)$ and using

$$\int_{-1}^{1} e^{i\epsilon\xi\eta}(1-\eta^2)^p d\eta = 2\cdot 2^p p!\,\frac{j_p(\epsilon\xi)}{(\epsilon\xi)^p}, \tag{61}$$

$$\int_{i\infty}^{1} e^{i\epsilon\xi\eta}(1-\eta^2)^p d\eta = 1\cdot 2^p p!\,\frac{h_p^{(1)}(\epsilon\xi)}{(\epsilon\xi)^p}, \tag{62}$$

$$\int_{-1}^{i\infty} e^{i\epsilon\xi\eta}(1-\eta^2)^p d\eta = 1\cdot 2^p p!\,\frac{h_p^{(2)}(\epsilon\xi)}{(\epsilon\xi)^p}, \tag{63}$$

where j_p, $h_p^{(1)}$, $h_p^{(2)}$ represent spherical Bessel and Hankel functions of the first and second kind, respectively. The series of spherical Bessel functions which results from the use of the first set of limits converges for all finite values of ξ and must therefore be proportional to ${}^{(1)}v_{lm}(\xi)$. Hence we obtain by evaluating

$${}^{(1)}v_{lm}(\xi) = \frac{q_{lm}}{2}\int_{-1}^{1} e^{i\epsilon\xi\eta}(1+\xi^2)^{m/2}(1-\eta^2)^{m/2}u_{lm}(\eta)d\eta \tag{64}$$

the expressions

$${}^{(1)}v_{lm}(\xi) = q_{lm}(1+\xi^2)^{m/2}\sum_{k=0}^{\infty} 2^{m+k}(m+k)!\,c_{2k}^{lm}\,\frac{j_{m+k}(\epsilon\xi)}{(\epsilon\xi)^{m+k}}, \quad (l-m) \text{ even}, \tag{65}$$

$${}^{(1)}v_{lm}(\xi) = iq_{lm}(1+\xi^2)^{m/2}\sum_{k=0}^{\infty} 2^{m+k}(m+k)!\,c_{2k}^{lm}\,\frac{j_{m+k+1}(\epsilon\xi)}{(\epsilon\xi)^{m+k}}, \quad (l-m) \text{ odd}. \tag{66}$$

The asymptotic behavior of these functions is obtained from the asymptotic expression for the Bessel functions

$$^{(1)}v_{lm}(\xi) \to q_{lm}(2/\epsilon)^m m!(\epsilon\xi)^{-1} \times \cos[\epsilon\xi - (m+1)\pi/2], \quad (l-m) \text{ even}, \quad (67)$$

$$^{(1)}v_{lm}(\xi) \to iq_{lm}(2/\epsilon)^m m!(\epsilon\xi)^{-1} \times \cos[\epsilon\xi - (m+2)\pi/2], \quad (l-m) \text{ odd}. \quad (68)$$

The joining factor q_{lm} is evaluated by comparing (65) and (66) with (32) and (33). One finds

$$q_{lm} = \left\{ \sum_{k=0}^{\infty} \frac{2^{m+k}(m+k)!c_{2k}^{lm}}{1\cdot 3\cdot 5\cdots[2(m+k)+1]} \right\}^{-1}, \quad (l-m) \text{ even}, \quad (69)$$

$$q_{lm} = -(i/\epsilon)\left\{ \sum_{k=0}^{\infty} \frac{2^{m+k}(m+k)!c_{2k}^{lm}}{1\cdot 3\cdot 5\cdots[2(m+k)+3]} \right\}^{-1}, \quad (l-m) \text{ odd}. \quad (70)$$

When the second and third sets of limits are applied to the radial function (59), series of Hankel functions as defined by (62) and (63) will result. Comparing the familiar relations

$$h_p^{(1)}(\epsilon\xi) = j_p(\epsilon\xi) + in_p(\epsilon\xi), \quad (71)$$

$$h_p^{(2)}(\epsilon\xi) = j_p(\epsilon\xi) - in_p(\epsilon\xi), \quad (72)$$

where n_p represents the spherical Neumann functions, with (57) and (58), leads one to conclude, first, that these limits produce the radial functions of the third and fourth kinds, respectively. Hence

$$^{(3),(4)}v_{lm}(\xi) = q_{lm}(1+\xi^2)^{m/2} \sum_{k=0}^{\infty} 2^{m+k}(m+k)!c_{2k}^{lm} \frac{h_{m+k}^{(1),(2)}(\epsilon\xi)}{(\epsilon\xi)^{m+k}}, \quad (l-m) \text{ even}, \quad (73)$$

$$^{(3),(4)}v_{lm}(\xi) = iq_{lm}(1+\xi^2)^{m/2} \sum_{k=0}^{\infty} 2^{m+k}(m+k)!c_{2k}^{lm} \frac{h_{m+k+1}^{(1),(2)}(\epsilon\xi)}{(\epsilon\xi)^{m+k}}, \quad (l-m) \text{ odd}. \quad (74)$$

Secondly, the functions of the second kind are represented by series of Neumann functions as follows

$$^{(2)}v_{lm}(\xi) = q_{lm}(1+\xi^2)^{m/2} \sum_{k=0}^{\infty} 2^{m+k}(m+k)!c_{2k}^{lm} \frac{n_{m+k}(\epsilon\xi)}{(\epsilon\xi)^{m+k}}, \quad (l-m) \text{ even}, \quad (75)$$

$$^{(3)}v_{lm}(\xi) = iq_{lm}(1+\xi^2)^{m/2} \sum_{k=0}^{\infty} 2^{m+k}(m+k)!c_{2k}^{lm} \frac{n_{m+k+1}(\epsilon\xi)}{(\epsilon\xi)^{m+k}}, \quad (l-m) \text{ odd}. \quad (76)$$

Since the spherical Neumann functions possess a singularity at the origin neither (73), (74) nor (75) and (76) converge well unless $\epsilon\xi$ is considerably greater than unity.

These functions are asymptotically given by

$$^{(2)}v_{lm}(\xi) \to q_{lm}(2/\epsilon)^m m!(\epsilon\xi)^{-1} \sin [\epsilon\xi - (m+1)\pi/2], \quad (l-m) \text{ even}, \tag{77}$$

$$^{(2)}v_{lm}(\xi) \to iq_{lm}(2/\epsilon)^m m!(\epsilon\xi)^{-1} \sin [\epsilon\xi - (m+2)\pi/2], \quad (l-m) \text{ odd}, \tag{78}$$

$$^{(3),(4)}v_{lm}(\xi) \to q_{lm}(2/\epsilon)^m m!(\mp i)^{m+1}(e^{\pm i\epsilon\xi})/\epsilon\xi, \quad (l-m) \text{ even}, \tag{79}$$

$$^{(3),(4)}v_{lm}(\xi) \to iq_{lm}(2/\epsilon)^m m!(\mp i)^{m+2}(e^{\pm i\epsilon\xi})/\epsilon\xi, \quad (l-m) \text{ odd}. \tag{80}$$

It is easily shown that the functions $^{(3),(4)}v_{lm}(\xi)$ represent spherically diverging and converging waves, at large distances.

From the asymptotic forms (67), (68), (77) and (78) for the functions $^{(1)}v_{lm}(\xi)$ and $^{(2)}v_{lm}(\xi)$ one obtains the asymptotic value of the Wronskian

$$W_{lm}(\xi) \to (-1)^{l-m}\epsilon^{-1}[q_{lm}(2/\epsilon)^m m!]^2/\xi^2. \tag{81}$$

Comparison of (81) with (36) determines one of the heretofore unknown quantities, namely,

$$W_{lm}(0) \to (-1)^{l-m}\epsilon^{-1}[q_{lm}(2/\epsilon)^m m!]^2. \tag{82}$$

There now remains to be found a value for the quantity C_{lm}, see (46) and (47). This, too, is determined from considerations of asymptotic behavior. Thus the asymptotic dependence of $^{(2)}v_{lm}(\xi)$ is given by (77) and (78). The asymptotic dependence of the functions $g_{lm}(\xi)$ is proportional to that of $d^{(1)}v_{lm}(\xi)/d\xi$ as follows from the differential equation (49), and by virtue of (67) and (68) this is proportional to the asymptotic form of $^{(2)}v_{lm}(\xi)$ also. Consequently C_{lm} must be so chosen that it be equal and opposite to the asymptotic value of $\tan^{-1}\xi$

$$C_{lm} = -\pi/2. \tag{83}$$

Finally, one can now write down specific values of Q_{lm}, cf. (47),

$$Q_{lm} = (-1)^{l-m}\epsilon^{-1}[q_{lm}(2/\epsilon)^m m!]^2\beta_{lm}; \tag{84}$$

for example,

$$Q_{l0} = \epsilon^{-1}q_{l0}^2 u_{l0}^2(0), \quad (l-m) \text{ even}, \tag{85}$$

$$Q_{l0} = \epsilon^{-1}q_{l0}^2 u_{l0}'^2(0), \quad (l-m) \text{ odd}, \tag{86}$$

$$Q_{l1} = 4\epsilon^{-3}q_{l1}^2 u_{l1}^2(0)(\tfrac{1}{2} - 2c_2^{l1}), \quad (l-m) \text{ even}, \tag{87}$$

$$Q_{l1} = 4\epsilon^{-3}q_{l1}^2 u_{l1}'^2(0)(\tfrac{3}{2} - 2c_2^{l1}), \quad (l-m) \text{ odd}, \tag{88}$$

and so on for higher integer values of m.

We now outline the connection between the oblate spheroidal functions defined by Stratton *et al.* (9), and those defined here. Since the associated Legendre functions corresponding to a given value of m constitute an orthogonal set in the interval $-1 \leqslant \eta \leqslant 1$ one may write

$$u_{lm}(\eta) = \sum_{n=0,1}^{\infty}{}' d_n^{lm} P_{m+n}^m(\eta), \tag{89}$$

where

$$d_n^{lm} = \frac{2m+2n+1}{2} \frac{n!}{(2m+n)!} \int_{-1}^{1} P_{m+n}^m(\eta) u_{lm}(\eta) d\eta. \tag{90}$$

Expressions for the radial functions which involve the same coefficients as in (89) may be obtained by inserting the expansion

$$(1-\eta^2)^{m/2} e^{i\epsilon\xi\eta} = \sum_{n=0}^{\infty} i^{m+n}(2m+2n+1) \frac{j_{m+n}(\epsilon\xi)}{(\epsilon\xi)^m} P_{m+n}^m(\eta) \tag{91}$$

in (64) and using (90). One finds

$${}^{(1)}v_{lm}(\xi) = q_{lm}(1+\xi^2)^{m/2} \sum_{n=0,1}^{\infty}{}' d_n^{lm} i^{m+n} \frac{(2m+n)!}{n!} \frac{j_{m+n}(\epsilon\xi)}{(\epsilon\xi)^m}. \tag{92}$$

The relation between the coefficients d_n^{lm} and c_{2k}^{lm} is found by inserting the expansions

$$2^{m+k} \frac{j_{m+k}(\epsilon\xi)}{(\epsilon\xi)^{m+k}} = 2^m k! \sum_{r=0}^{\infty} \frac{(m+2r+1/2)\Gamma(m+r+1/2)}{r!\Gamma(k-r+1)\Gamma(m+k+r+3/2)} \frac{j_{m+2r}(\epsilon\xi)}{(\epsilon\xi)^m}, \tag{93}$$

$$2^{m+k} \frac{j_{m+k}(\epsilon\xi)}{(\epsilon\xi)^{m+k}} = 2^m k! \sum_{r=0}^{\infty} \frac{(m+2r+3/2)\Gamma(m+r+3/2)}{r!\Gamma(k-r+1)\Gamma(m+k+r+5/2)} \frac{j_{m+2r+1}(\epsilon\xi)}{(\epsilon\xi)^m} \tag{94}$$

in (65) and (66), respectively. One finds

$$i^{m+2r} \frac{(2m+2r)!}{(2r)!} d_{2r}^{lm} = 2^m (m+2r+1/2)(r!)^{-1}\Gamma(m+r+1/2) \times \sum_{k=0}^{\infty} \frac{k!(k+m)!c_{2k}^{lm}}{\Gamma(k-r+1)\Gamma(m+k+r+3/2)}, \tag{95}$$

$$i^{m+2r+1} \frac{(2m+2r+1)!}{(2r+1)!} d_{2r+1}^{lm} = 2^m (m+2r+3/2)(r!)^{-1}\Gamma(m+r+3/2) \times \sum_{k=0}^{\infty} \frac{k!(k+m)!c_{2k}^{lm}}{\Gamma(k-r+1)\Gamma(m+k+r+5/2)}. \tag{96}$$

Since (93) and (94) are valid if the spherical Bessel functions are replaced with either spherical Neumann or Hankel functions it follows that the radial functions of the second, third and fourth kinds may be formed by replacing the spherical Bessel functions in (92) by the spher-

ical Neumann or Hankel functions, respectively. If the angular function (89) is multiplied by $(l+m)!/2^m m!\,(l-m)!$ it becomes identical with that of Stratton but the radial functions defined by expansions of the form of (92) have a normalization different from that used by Stratton. To obtain Stratton's radial functions one must divide our radial functions by

$$q_{lm}(2/\epsilon)^m m!(-1)^{(l-m)/2}, \qquad \text{when } (l-m) \text{ is even,} \tag{97}$$

$$iq_{lm}(2/\epsilon)^m m!(-1)^{(l-m-1)/2}, \qquad \text{when } (l-m) \text{ is odd.} \tag{98}$$

4. EXPANSIONS FOR SPHERICAL AND PLANE WAVES

We present useful expansions in terms of spheroidal eigenfunctions ψ_{lm}, cf. (7), especially for functions important in solutions to wave problems.

Consider first Green's function of the scalar wave equation (4) for the infinite domain, in three dimensions, $e^{ikR}/4\pi R$; here R measures the distance between the "source" point $Q(\xi', \eta', \varphi'$ *or* $r', \theta', \varphi')$ and the point "of observation" $P(\xi, \eta, \varphi$ *or* $r, \theta, \varphi)$. The general theory for a Green's function $G(P; Q)$ gives the relation

$$G(P;Q) = \sum_n y_{1n}(x_1)y_{1n}(x_1')y_{2n}(x_2)y_{2n}(x_2')g_n(x_3;x_3'). \tag{99}$$

The y_{jn}, $j = 1, 2, 3$ are the n'th eigenfunctions, normalized to unity, corresponding to the j'th coordinates of a three dimensional orthogonal system in which the operator ∇^2 is separable; and g_n denotes the Green's function corresponding to the ordinary equation in x_3 at the n'th eigenvalue. Relation (99) reduces the problem of finding G to one dimension.

In our case we note first that $R = (r^2 + r'^2 - 2rr'\cos\gamma)^{\frac{1}{2}}$, where γ is the angle between $\mathbf{r}$ and $\mathbf{r}'$ and $\cos\gamma = \sin\theta\sin\theta'\cos(\varphi-\varphi') + \cos\theta\cos\theta'$. Hence our function is not only symmetric in φ, φ' but a function of $(\varphi - \varphi')$, and may be expanded with use of the eigenfunctions $\cos m(\varphi-\varphi')$. Thus

$$\frac{e^{ikR}}{4\pi R} = \sum_{l=0}^{\infty}\left[\frac{u_{l0}(\eta)u_{l0}(\eta')}{N_{l0}}\frac{1}{2\pi}g_{l0}(\xi;\xi') + \sum_{m=1}^{l}\frac{u_{lm}(\eta)u_{lm}(\eta')}{N_{lm}}\frac{\cos m(\varphi-\varphi')}{\pi}g_{lm}(\xi;\xi')\right]. \tag{100}$$

Drawing again from the general theory, one knows that $g_{lm}(\xi;\xi')$ satisfies

$$\frac{d}{d\xi}\left[(1+\xi^2)\frac{dg_{lm}(\xi;\xi')}{d\xi}\right] + \left[\frac{m^2}{1+\xi^2} - \alpha_{lm} + \epsilon^2(1+\xi^2)\right]g_{lm}(\xi;\xi') = -\delta(\xi-\xi'). \tag{101}$$

When (101) is integrated with respect to ξ over an interval including ξ', and one lets the interval approach zero, one obtains

$$(1 + \xi'^2)\{[dg_{lm}(\xi;\xi')/d\xi]_{\xi=\xi'+0} - [dg_{lm}(\xi;\xi')/d\xi]_{\xi=\xi'-0}\} = -1, \quad (102)$$

for one may assume $dg_{lm}/d\xi \gg g_{lm}$, since g_{lm} or its derivative possesses a discontinuity at $\xi = \xi'$. In fact (102) *asserts* that $\frac{dg_{lm}}{d\xi}$ possesses the discontinuity $-1/(1 + \xi'^2)$ there. But by (36) and (82)

$$\begin{aligned} -1/(1+\xi'^2) &= -W_{lm}(\xi')/W_{lm}(0) \\ &= (-1)^{l-m+1}\epsilon[q_{lm}(2/\epsilon)^m m!]^{-2}(-i)[{}^{(1)}v_{lm}(\xi')\,{}^{(3)}v'_{lm}(\xi') \\ &\qquad - {}^{(1)}v'_{lm}(\xi')\,{}^{(3)}v_{lm}(\xi')], \end{aligned} \quad (103)$$

from which g_{lm} can be obtained. We choose ${}^{(3)}v_{lm}$ because, as is easily shown

$$e^{ikR}/4\pi R = \begin{cases} (e^{ikr}/4\pi r)e^{-ikr'\cos\gamma}, & \text{when} \quad r \gg r', \\ (e^{ikr'}/4\pi r')e^{-ikr\cos\gamma}, & \text{when} \quad r' \gg r, \end{cases} \quad (104)$$

and therefore the radial Green's function must behave as a spherically diverging wave as either r or $r' \to \infty$. Thus, when normalized such that $g_{lm} \equiv e^{ikr}/r$ at large ξ (or ξ')

$$g_{lm}(\xi;\xi') = i(-1)^{l-m}k[g_{lm}(2/\epsilon)^m m!]^{-2}\begin{cases} {}^{(1)}v_{lm}(\xi)\,{}^{(3)}v_{lm}(\xi'), & \xi \leqslant \xi', \\ {}^{(1)}v_{lm}(\xi')\,{}^{(3)}v_{lm}(\xi), & \xi \geqslant \xi', \end{cases} \quad (105)$$

and

$$\begin{aligned} e^{ikR}/kR = \sum_{l=0}^{\infty}\Bigg\{ &\frac{2i(-1)^l}{N_{l0}q_{l0}^2}u_{l0}(\eta)u_{l0}(\eta')\begin{bmatrix} {}^{(1)}v_{l0}(\xi)\,{}^{(3)}v_{l0}(\xi') \\ {}^{(1)}v_{l0}(\xi')\,{}^{(3)}v_{l0}(\xi) \end{bmatrix} \\ &+ 2\sum_{m=1}^{l}\frac{2i(-1)^{l-m}}{N_{lm}[q_{lm}(2/\epsilon)^m m!]^2}u_{lm}(\eta)u_{lm}(\eta') \\ &\times\begin{bmatrix} {}^{(1)}v_{lm}(\xi)\,{}^{(3)}v_{lm}(\xi') \\ {}^{(1)}v_{lm}(\xi')\,{}^{(3)}v_{lm}(\xi) \end{bmatrix}\cos m(\varphi - \varphi')\Bigg\}. \end{aligned} \quad (106)$$

If the point Q is moved to infinity along a fixed direction η', φ', equation (97) leads directly to an expansion, in terms of eigenfunctions, for a plane wave progressing along the specified direction, that is,

$$\begin{aligned} e^{-ikr\cos\gamma} = \sum_{l=0}^{\infty}\Bigg\{ &\frac{2(-1)^l}{N_{l0}q_{l0}}u_{l0}(\eta)u_{l0}(\eta')\,{}^{(1)}v_{l0}(\xi) + 2\sum_{m=1}^{l}\frac{2^{1-m}(-1)^l i^m\epsilon^m}{N_{lm}q_{lm}m!} \\ &\times u_{lm}(\eta)u_{lm}(\eta')\,{}^{(1)}v_{lm}(\xi)\cos m(\varphi - \varphi')\Bigg\}. \end{aligned} \quad (107)$$

In particular, when the plane wave is propagated parallel to the z axis, $\eta' = \pm 1$ and $r\cos\gamma = z$

$$e^{\mp ikz} = \sum_{l=0}^{\infty}\frac{2(\mp 1)^l}{N_{l0}q_{l0}}u_{l0}(\eta)\,{}^{(1)}v_{l0}(\xi). \quad (108)$$

5. TABLES AND GRAPHS

Table I lists eigenvalues α_{lm} as defined in (15). Since eigenvalues for $m = 0$, and α_{11}, α_{21}, α_{31}, α_{32}, and α_{33} have been given previously by Stratton *et al.* (9) and/or Bouwkamp (10), they are omitted here. In comparing their work with that given here one must keep in mind the following differences in notation: (*a*) our $(l - m)$ corresponds to Stratton's l; (*b*) our ϵ to Stratton's c; and (*c*) our $\epsilon^2 - \alpha$ to Stratton's B.

The values in our tables are presumably accurate to 0.5 of the last place.

TABLE I.—α_{lm}.

	$\epsilon=1$	$\epsilon=2$	$\epsilon=3$	$\epsilon=4$	$\epsilon=5$
α_{41}	20.520683	22.0977	24.7772	28.628718	33.694934
α_{51}	30.513714	32.065706	34.689634	38.446240	43.439511
α_{61}	42.509762	44.047142	46.63676	50.321044	55.161310
α_{71}	56.507305	58.035455	60.603297	64.243436	69.000355
α_{81}	72.505673	74.027623	76.580708	80.189948	84.890953
α_{22}	6.855163	9.3950	13.527	19.028	25.431
α_{42}	20.597220	22.3883	25.386	29.673	35.512
α_{52}	30.564371	32.2612	35.105	39.130	44.384
α_{62}	42.545807	44.1876	46.940	50.831	55.911
α_{72}	56.534275	58.141236	60.83385	64.6355	69.582
α_{82}	72.526617	74.110123	76.76175	80.5011	85.3572
α_{92}	90.521270	92.088251	94.710588	98.40481	103.1950
α_{43}	20.72588	22.88641	26.42748	31.2495	37.1923
α_{53}	30.649090	32.59105	35.814	40.318	46.162
α_{63}	42.605978	44.42319	47.45149	51.6976	57.186
α_{73}	56.579265	58.31813	61.22080	65.2975	70.569
α_{83}	72.561542	74.24790	77.06476	81.022	86.142
α_{93}	90.549163	92.19841	94.95326	98.82386	103.8261
$\alpha_{10,3}$	110.540200	112.16268	114.87324	118.68203	123.6044
α_{44}	20.90851	23.62673	28.1304	34.370	42.247
α_{54}	30.76841	33.06243	36.8468	42.057	48.590
α_{64}	42.690428	44.75609	48.1814	52.946	59.044
α_{74}	56.642333	58.567048	61.76832	66.2401	71.983
α_{84}	72.610475	74.441365	77.49170	81.7620	87.2575
α_{94}	90.588260	92.353398	95.296855	99.42228	104.7374
$\alpha_{10,4}$	110.572146	112.289375	115.154293	119.17199	124.3511
$\alpha_{11,4}$	132.560081	134.241314	137.046810	140.98222	146.0564

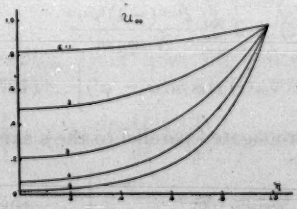


FIG. 2.

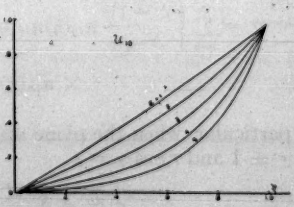


FIG. 3.

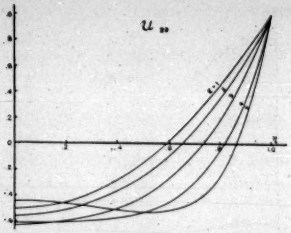


FIG. 4.

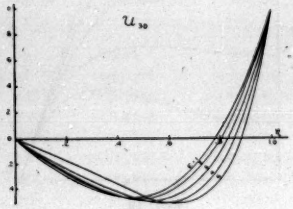
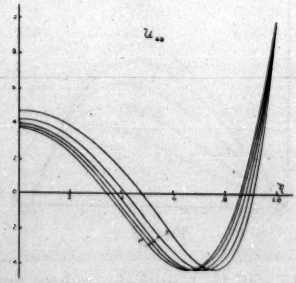


FIG. 5.



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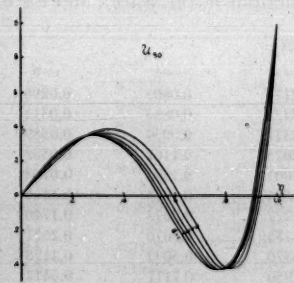

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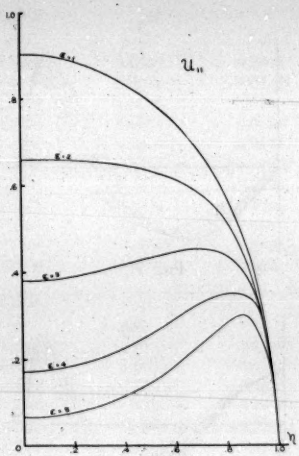

FIG. 8.

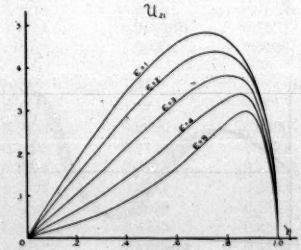

FIG. 9.

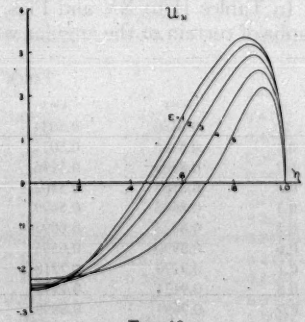

FIG. 10.

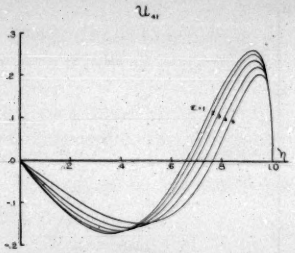

FIG. 11.

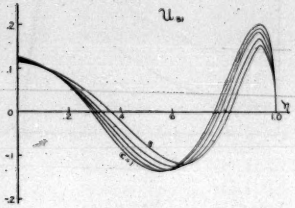

FIG. 12.

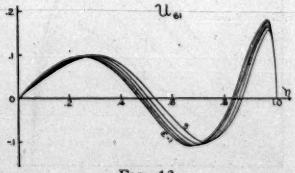

FIG. 13.

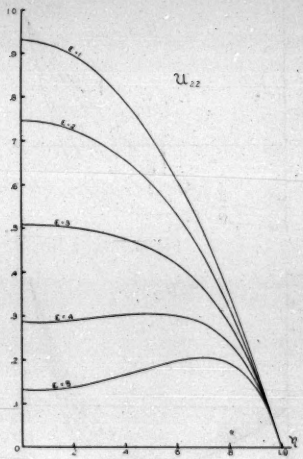

FIG. 14.

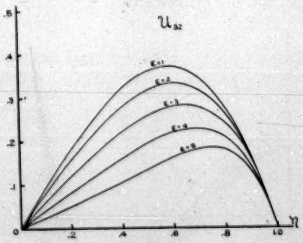

FIG. 15.

In Tables II to XV and Figs. 2 to 15 will be found the values and graphs of certain of the angular wave functions $u_{10}(\eta)$, $u_{11}(\eta)$, and $u_{12}(\eta)$.

TABLE II.—$u_{00}(\eta)$.

η	$\epsilon=1$	$\epsilon=2$	$\epsilon=3$	$\epsilon=4$	$\epsilon=5$
0	0.8450	0.5034	0.2179	0.0805	0.0290
0.1	0.8465	0.5074	0.2227	0.0843	0.0314
0.2	0.8509	0.5196	0.2372	0.0959	0.0389
0.3	0.8583	0.5402	0.2625	0.1167	0.0532
0.4	0.8688	0.5699	0.3001	0.1493	0.0768
0.5	0.8823	0.6093	0.3524	0.1975	0.1146
0.6	0.8990	0.6595	0.4227	0.2673	0.1740
0.7	0.9190	0.7218	0.5155	0.3670	0.2671
0.8	0.9424	0.7979	0.6370	0.5092	0.4128
0.9	0.9694	0.8898	0.7950	0.7117	0.6412
1.0	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE III.—$u_{10}(\eta)$.

η	$\epsilon=1$	$\epsilon=2$	$\epsilon=3$	$\epsilon=4$	$\epsilon=5$
0	0.0000	0.0000	0.0000	0.0000	0.0000
0.1	0.0906	0.0677	0.0425	0.0231	0.0115
0.2	0.1818	0.1371	0.0874	0.0489	0.0251
0.3	0.2740	0.2099	0.1376	0.0802	0.0433
0.4	0.3679	0.2881	0.1961	0.1205	0.0697
0.5	0.4641	0.3736	0.2665	0.1747	0.1093
0.6	0.5631	0.4686	0.3531	0.2492	0.1700
0.7	0.6655	0.5756	0.4617	0.3531	0.2641
0.8	0.7720	0.6976	0.5991	0.4993	0.4107
0.9	0.8833	0.8377	0.7746	0.7062	0.6400
1.0	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE IV.—$u_{20}(\eta)$.

η	$\epsilon=1$	$\epsilon=2$	$\epsilon=3$	$\epsilon=4$	$\epsilon=5$
0	−0.5129	−0.5599	−0.6283	−0.6097	−0.4419
0.1	−0.4988	−0.5484	−0.6211	−0.6089	−0.4473
0.2	−0.4565	−0.5135	−0.5989	−0.6056	−0.4626
0.3	−0.3854	−0.4538	−0.5589	−0.5964	−0.4856
0.4	−0.2851	−0.3670	−0.4966	−0.5750	−0.5108
0.5	−0.1546	−0.2498	−0.4050	−0.5312	−0.5282
0.6	0.00713	−0.0977	−0.2743	−0.4489	−0.5187
0.7	0.2016	0.0951	−0.0906	−0.3030	−0.4478
0.8	0.4305	0.3359	0.1647	−0.0559	−0.2545
0.9	0.6959	0.6339	0.4831	0.3497	0.1672
1.0	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE V.—$u_{30}(\eta)$.

η	$\epsilon=1$	$\epsilon=2$	$\epsilon=3$	$\epsilon=4$	$\epsilon=5$
0	0.0000	0.0000	0.0000	0.0000	0.0000
0.1	−0.1460	−0.1409	−0.1305	−0.1126	−0.0873
0.2	−0.2778	−0.2702	−0.2534	−0.2222	−0.1760
0.3	−0.3812	−0.3757	−0.3599	−0.3244	−0.2663
0.4	−0.4416	−0.4443	−0.4390	−0.4080	−0.3553
0.5	−0.4440	−0.4606	−0.4762	−0.4724	−0.4346
0.6	−0.3726	−0.4070	−0.4515	−0.4846	−0.4849
0.7	−0.2106	−0.2624	−0.3374	−0.4147	−0.4689
0.8	0.0593	−0.0014	−0.0958	−0.2083	−0.3161
0.9	0.4561	0.4068	0.3260	0.2194	0.0995
1.0	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE VI.—$u_{40}(\eta)$.

η	$\epsilon=1$	$\epsilon=2$	$\epsilon=3$	$\epsilon=4$	$\epsilon=5$
0	0.3775	0.3854	0.4006	0.4279	0.4752
0.1	0.3411	0.3532	0.3696	0.4015	0.4546
0.2	0.2369	0.2520	0.2793	0.3235	0.3924
0.3	0.0797	0.1007	0.1383	0.1977	0.2874
0.4	−0.1053	−0.0812	−0.0374	0.0328	0.1402
0.5	−0.2827	−0.2621	−0.2221	−0.1537	−0.0429
0.6	−0.4059	−0.3975	−0.3760	−0.3298	−0.2413
0.7	−0.4167	−0.4283	−0.4396	−0.4381	−0.4049
0.8	−0.2445	−0.2771	−0.3255	−0.3800	−0.4264
0.9	0.1949	0.1562	0.0935	0.0093	−0.0928
1.0	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE VII.—$u_{30}(\eta)$.

η	$\epsilon=1$	$\epsilon=2$	$\epsilon=3$	$\epsilon=4$	$\epsilon=5$
0	0.0000	0.0000	0.0000	0.0000	0.0000
0.1	0.1782	0.1763	0.1729	0.1676	0.1593
0.2	0.3073	0.3065	0.3046	0.3006	0.2922
0.3	0.3471	0.3520	0.3592	0.3669	0.3715
0.4	0.2754	0.2893	0.3117	0.3403	0.3710
0.5	0.0973	0.1196	0.1568	0.2079	0.2698
0.6	−0.1449	−0.1210	−0.0793	−0.0179	0.0635
0.7	−0.3612	−0.3480	−0.3219	−0.2776	−0.2096
0.8	−0.4030	−0.4120	−0.4224	−0.4275	−0.4185
0.9	−0.0505	−0.0781	−0.1220	−0.1788	−0.2436
1.0	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE VIII.—$u_{11}(\eta)$.

η	$\epsilon=1$	$\epsilon=2$	$\epsilon=3$	$\epsilon=4$	$\epsilon=5$
0	0.9041	0.6612	0.3807	0.1703	0.0638
0.1	0.9005	0.6608	0.3831	0.1736	0.0665
0.2	0.8894	0.6594	0.3902	0.1837	0.0750
0.3	0.8704	0.6561	0.4015	0.2006	0.0898
0.4	0.8422	0.6498	0.4160	0.2246	0.1134
0.5	0.8032	0.6382	0.4321	0.2551	0.1432
0.6	0.7503	0.6176	0.4462	0.2908	0.1838
0.7	0.6786	0.5820	0.4523	0.3269	0.2326
0.8	0.5788	0.5200	0.4374	0.3519	0.2810
0.9	0.4277	0.4045	0.3702	0.3320	0.2970
1.0	0.0000	0.0000	0.0000	0.0000	0.0000

TABLE IX.—$u_{21}(\eta)$.

η	$\epsilon=1$	$\epsilon=2$	$\epsilon=3$	$\epsilon=4$	$\epsilon=5$
0	0.0000	0.0000	0.0000	0.0000	0.0000
0.1	0.0927	0.0747	0.0525	0.0321	0.0171
0.2	0.1830	0.1489	0.1056	0.0660	0.0366
0.3	0.2682	0.2206	0.1597	0.1024	0.0595
0.4	0.3453	0.2885	0.2145	0.1433	0.0879
0.5	0.4104	0.3498	0.2692	0.1891	0.1238
0.6	0.4586	0.4004	0.3210	0.2390	0.1685
0.7	0.4821	0.4329	0.3638	0.2893	0.2212
0.8	0.4679	0.4338	0.3844	0.3284	0.2737
0.9	0.3870	0.3720	0.3494	0.3222	0.2938
1.0	0.0000	0.0000	0.0000	0.0000	0.0000

TABLE X.—$u_{31}(\eta)$.

η	$\epsilon=1$	$\epsilon=2$	$\epsilon=3$	$\epsilon=4$	$\epsilon=5$
0	−0.2474	−0.2419	−0.2390	−0.2399	−0.2237
0.1	−0.2344	−0.2308	−0.2306	−0.2346	−0.2216
0.2	−0.1961	−0.1980	−0.2054	−0.2182	−0.2150
0.3	−0.1342	−0.1442	−0.1631	−0.1894	−0.2018
0.4	−0.0522	−0.0714	−0.1036	−0.1463	−0.1786
0.5	0.0447	0.0171	−0.0277	−0.0866	−0.1407
0.6	0.1487	0.1157	0.0624	−0.0086	−0.0821
0.7	0.2476	0.2144	0.1603	0.0864	0.0024
0.8	0.3210	0.2945	0.2505	0.1882	0.1114
0.9	0.3279	0.3147	0.2922	0.2589	0.2146
1.0	0.0000	0.0000	0.0000	0.0000	0.0000

TABLE XI.—$u_{41}(\eta)$.

η	$\epsilon=1$	$\epsilon=2$	$\epsilon=3$	$\epsilon=4$	$\epsilon=5$
0	0.0000	0.0000	0.0000	0.0000	0.0000
0.1	−0.0715	−0.0677	−0.0617	−0.0539	−0.0443
0.2	−0.1311	−0.1250	−0.1153	−0.1023	−0.0858
0.3	−0.1676	−0.1620	−0.1528	−0.1390	−0.1211
0.4	−0.1717	−0.1698	−0.1660	−0.1586	−0.1452
0.5	−0.1370	−0.1416	−0.1476	−0.1520	−0.1506
0.6	−0.0619	−0.0739	−0.0918	−0.1116	−0.1277
0.7	0.0476	0.0298	0.0023	−0.0312	−0.0651
0.8	0.1714	0.1530	0.1238	0.0860	0.0433
0.9	0.2582	0.2471	0.2289	0.2041	0.1737
1.0	0.0000	0.0000	0.0000	0.0000	0.0000

TABLE XII.—$u_{51}(\eta)$.

η	$\epsilon=1$	$\epsilon=2$	$\epsilon=3$	$\epsilon=4$	$\epsilon=5$
0	0.1245	0.1231	0.1212	0.1195	0.1191
0.1	0.1070	0.1067	0.1064	0.1068	0.1088
0.2	0.0588	0.0610	0.0649	0.0707	0.0788
0.3	−0.0090	−0.0038	−0.0050	0.0167	0.0324
0.4	−0.0829	−0.0717	−0.0606	−0.0449	−0.0241
0.5	−0.1269	−0.1218	−0.1129	−0.0992	−0.0795
0.6	−0.1319	−0.1317	−0.1302	−0.1258	−0.1161
0.7	−0.0753	−0.0818	−0.0912	−0.1012	−0.1090
0.8	0.0443	0.0331	0.0155	−0.0068	−0.0314
0.9	0.1843	0.1754	0.1607	0.1409	0.1164
1.0	0.0000	0.0000	0.0000	0.0000	0.0000

TABLE XIII.—$u_{61}(\eta)$.

η	$\epsilon=1$	$\epsilon=2$	$\epsilon=3$	$\epsilon=4$	$\epsilon=5$
0	0.0000	0.0000	0.0000	0.0000	0.0000
0.1	0.0577	0.0566	0.0544	0.0514	0.0477
0.2	0.0932	0.0923	0.0900	0.0865	0.0823
0.3	0.0910	0.0922	0.0926	0.0926	0.0923
0.4	0.0478	0.0521	0.0573	0.0636	0.0710
0.5	−0.0230	−0.0165	−0.0074	0.0045	0.0193
0.6	−0.0909	−0.0850	−0.0765	−0.0645	−0.0483
0.7	−0.1119	−0.1104	−0.1087	−0.1051	−0.0979
0.8	−0.0435	−0.0483	−0.0565	−0.0664	−0.0762
0.9	0.1127	0.1060	0.0948	0.0797	0.0614
1.0	0.0000	0.0000	0.0000	0.0000	0.0000

TABLE XIV.—$u_{22}(\eta)$.

η	$\epsilon=1$	$\epsilon=2$	$\epsilon=3$	$\epsilon=4$	$\epsilon=5$
0	0.9307	0.7465	0.5079	0.2856	0.1301
0.1	0.9221	0.7413	0.5066	0.2870	0.1324
0.2	0.8961	0.7254	0.5021	0.2908	0.1393
0.3	0.8525	0.6980	0.4936	0.2964	0.1505
0.4	0.7909	0.6580	0.4793	0.3020	0.1654
0.5	0.7107	0.6035	0.4561	0.3049	0.1823
0.6	0.6113	0.5320	0.4202	0.3008	0.1980
0.7	0.4917	0.4404	0.3660	0.2826	0.2059
0.8	0.3509	0.3248	0.2856	0.2393	0.1935
0.9	0.1874	0.1800	0.1684	0.1539	0.1383
1.0	0.0000	0.0000	0.0000	0.0000	0.0000

TABLE XV.—$u_{22}(\eta)$.

η	$\epsilon=1$	$\epsilon=2$	$\epsilon=3$	$\epsilon=4$	$\epsilon=5$
0	0.0000	0.0000	0.0000	0.0000	0.0000
0.1	0.0937	0.0793	0.0600	0.0403	0.0242
0.2	0.1820	0.1549	0.1182	0.0806	0.0493
0.3	0.2595	0.2228	0.1725	0.1204	0.0760
0.4	0.3207	0.2786	0.2202	0.1585	0.1045
0.5	0.3597	0.3174	0.2576	0.1928	0.1339
0.6	0.3706	0.3332	0.2792	0.2188	0.1615
0.7	0.3470	0.3189	0.2773	0.2291	0.1810
0.8	0.2823	0.2660	0.2413	0.2114	0.1798
0.9	0.1692	0.1640	0.1558	0.1455	0.1340
1.0	0.0000	0.0000	0.0000	0.0000	0.0000

The norms N_{lm} of these functions, cf. (28), (29), are listed in Table XVI.

TABLE XVI.

	$\epsilon=1$	$\epsilon=2$	$\epsilon=3$	$\epsilon=4$	$\epsilon=5$
N_{00}	1.6086	0.9013	0.4713	0.3019	0.2269
N_{10}	0.6167	0.5000	0.3785	0.2863	0.2247
N_{20}	0.3998	0.4181	0.4900	0.5357	0.4339
N_{30}	0.2856	0.2861	0.2860	0.2789	0.2571
N_{40}	0.2222	0.2223	0.2234	0.2286	0.2464
N_{50}	0.1819	0.1818	0.1842	0.1815	0.1849
N_{11}	1.1361	0.7041	0.3282	0.1352	0.0649
N_{21}	0.2459	0.1946	0.1359	0.0882	0.0566
N_{31}	0.0914	0.0819	0.0712	0.0644	0.0587
N_{41}	0.0434	0.0404	0.0363	0.0318	0.0270
N_{51}	0.0238	0.0227	0.0211	0.0192	0.0176
N_{61}	0.0145	0.0140	0.0132	0.0123	0.0113

Of the constants associated with the radial functions we list here certain values of q_{lm} and Q_{lm}.

TABLE XVII.

	$\epsilon=1$	$\epsilon=2$	$\epsilon=3$	$\epsilon=4$	$\epsilon=5$
q_{00}	1.1165	1.5251	2.3070	3.3216	4.3716
q_{10}	$-(3.1202)i$	$-(1.7407)i$	$-(1.3568)i$	$-(1.2100)i$	$-(1.1444)i$
q_{20}	−43.729	−10.213	−4.494	−3.079	−3.028
q_{30}	$(176.8)i$	$(22.88)i$	$(7.359)i$	$(3.588)i$	$(2.279)i$
q_{40}	1.10×10^{4}	672.0	128.0	38.5	15.0
q_{50}	$-(4.39\times10^{4})i$	$-(1.39\times10^{3})i$	$-(187.0)i$	$-(46.5)i$	$-(16.4)i$
q_{11}	1.6254	2.0735	3.1128	5.2787	8.9943
q_{21}	$-(7.811)i$	$-(4.399)i$	$-(3.543)i$	$-(3.380)i$	$-(3.532)i$
q_{31}	−260.9	−66.82	−27.320	−15.160	−11.011
q_{41}	$(1,112.0)i$	$(143.0)i$	$(44.73)i$	$(20.68)i$	$(12.35)i$
q_{51}	1.10×10^{5}	6.62×10^{3}	1.31×10^{3}	405.0	161.0
q_{61}	$-(4.47\times10^{5})i$	$-(1.229\times10^{4})i$	$-(1.90\times10^{3})i$	$-(477.7)i$	$-(161.5)i$
Q_{00}	0.890142	0.294712	0.084214	0.019511	0.0032136
Q_{10}	−7.975916	−0.688485	−0.108498	0.018937	0.0032491
Q_{20}	5.0300×10^{2}	1.63515×10^{1}	2.65847	0.93118	0.35806
Q_{30}	-6.8719×10^{4}	-5.3346×10^{2}	-3.1337×10^{1}	−4.1107	−0.78718
Q_{40}	1.73×10^{7}	3.555×10^{4}	8.725×10^{2}	6.794	1.2757
Q_{50}	6.71×10^{9}	3.268×10^{6}	3.789×10^{4}	1.622×10^{3}	1.437×10^{2}

Acknowledgments

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Phosphorus Removal from Sewage.—Two University of Wisconsin engineers, Profs. William L. Lea and Gerard A. Rohlich of the department of civil engineering, announced recently that they have developed a pilot plant which looks like it can take the phosphorus out of sewage effluent.

Large-scale use of the discovery would mean: (1) Sewage effluent would no longer contaminate lakes and streams with surplus phosphorus, one of the elements held responsible for nuisance growths of algae; (2) Sewage phosphorus, which up to now has been lost, could be refined and sold as commercial fertilizer; and (3) Atomic-research men would no longer have to worry about the danger from radio-active phosphorus in wastes.

The engineers made their progress report at a special meeting of the University's Lake Investigations committee. Lea and Rohlich have been working on the project for two years under grants from the committee and from the National Institute of Health.

Working experimentally in the U. W. hydraulics laboratory and at the Nine Springs Treatment plant of the Madison Metropolitan Sewerage district, they have been able to remove 99 per cent of the phosphorus from the effluent. The ordinary sewage effluent being discharged by the Nine Springs plant contains about 5 parts-per-million of phosphorus. Lea and Rohlich have brought this level down to 7-tenths parts-per-million. This level is actually below the amount of phosphorus contained in most natural creek water.

Lea and Rohlich estimate they could get about a ton of salable phosphorus fertilizer a day out of the 15 million gallons of sewage being processed daily at the Nine Springs plant. They can also retrieve most of the chemicals used in removing the phosphorus. So the net cost of materials for removing the phosphorus at the Nine Springs plant would be less than $90 a day, according to their figures. Lea and Rohlich estimate that the cost of constructing a de-phosphorizing plant at Nine Springs would be about $1,500,000.

The Lea-Rohlich process not only removes virtually all the phosphorus from sewage effluent, but it also cuts down the organic nitrogen by 60 per cent and the "biochemical oxygen demand" by 84 per cent. These are two other factors which help cause excessive algal growths in lakes into which sewage effluent is being dumped. The Lea-Rohlich process has no apparent effect on the amount of ammonia, nitrates, and nitrites in sewage effluent—three other factors favorable to algal growth.

Nevertheless, U. W. scientists see the Lea-Rohlich de-phosphorizing process as the first significant advance in what they call "tertiary sewage treatment," and one of the first successful attempts to recapture and re-use valuable materials which have been "going down the river."

Typewriter Ribbons (*du Pont Magazine*, Vol. 44, No. 1).—Typewriter ribbons are now made of nylon. This fiber's strength and resiliency withstand the severe pounding of typewriter keys, especially those of electric machines, much longer than do other types. And they produce sharp, clear impressions because nylon's strength makes possible a thinner ribbon. The smooth surface of the fabric prevents lint from clogging the type faces. These ribbons are manufactured by Remington Rand, Inc., Bridgeport, Conn.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS*

SYNTHETIC MICA

Synthetic mica, with essentially the same properties as natural mica, but able to withstand much higher temperatures, has now been crystallized successfully by Dr. Herbert Insley, Alvin Van Valkenburg, and Robert Pike at the National Bureau of Standards. This work is part of a broad program of fundamental research on fluorine-type artificial minerals carried on by the National Bureau of Standards under the sponsorship of the Office of Naval Research. The synthetic mica phase of the program has been carried out in cooperation with the U. S. Bureau of Mines and the Colorado School of Mines.

In the Bureau's work on synthetic mica the requirements of eventual quantity production have always been kept in view. Mica could be synthesized by duplicating the conditions under which it is formed in nature, but this would involve extremely high pressures as well as high temperatures. For safety and convenience it is better to work at atmospheric pressure. For that reason the Bureau's scientists are using fluorine as a crystallizing agent to grow crystals of mica without using high pressure. Natural fluorine is a gas, poisonous and difficult to control, but a group of synthetic fluorine compounds, the fluorosilicates, provides a convenient way of introducing fluorine into mica synthesis.

The raw materials for making synthetic mica are similar to the raw materials sometimes used in making glass: quartz, magnesite, bauxite, and a fluorosilicate compound (the only unusual ingredient). The raw mixture is placed in a platinum-lined crucible and melted in an electric furnace at a temperature of nearly 1400 degrees Centigrade. As the furnace cools, mica crystals grow from a tiny seed at the bottom of the crucible.

The most satisfactory synthetic mica developed so far has the chemical formula $K_4Mg_{12}Al_2Si_{12}O_{40}F_8$. This is equivalent to a form of natural mica in which the hydroxyl radical has been replaced by fluorine. Impurities may occur in the synthetic mica in the form of milky films parallel to the individual layers or white patches between crystals. Crystals free of impurities are clear and transparent, and thin flakes are easily split away along the planes of natural cleavage. The synthetic form has physical properties which compare favorably with natural mica. Electrical measurements on several clear flakes, $\frac{3}{4}$ in. square by $\frac{1}{32}$ in. thick, indicate a dielectric constant of about 6.3. The largest

* Communicated by the Director.

crystals grown so far at the National Bureau of Standards have a surface area of 4 sq. in.

Satisfactory mica synthesis depends to a large extent on the materials used in the crucible lining. Ceramic linings are badly corroded by a fluorine-bearing melt. Carbon and silicon carbide crucibles are somewhat better, but fluoride gases escape through the relatively porous walls, and fine carbon particles become imbedded throughout the synthesized mica. Crucibles lined with platinum foil give the best results. They do not react with the melt and are able to withstand high temperatures for long periods. Although initial cost is high, the platinum linings may be melted down and re-formed again and again.

The shape of the crucible is also important in mica synthesis. Flat-bottomed crucibles are undesirable because they offer a large surface for the formation of many seed crystals which grow independently in different directions and thus limit the development of single large crystals. Mica has a sheetlike structure and grows faster in a direction parallel to its cleavage plane than in any other direction. Consequently, if a crucible with a cone-shaped bottom is used, the number of seed crystals is reduced and the direction of growth tends to be upward. Experiments are now in progress to find ways of predetermining crystal growth even more completely in order to grow large parallel sheets. The key to this seems to be precise control of temperature differences within the crucible.

Mica's remarkable properties as an electrical insulator have made it invaluable to electrical and electronics industries. The United States is the world's largest consumer of natural mica, but it produces only enough to meet a third of its requirements. During 1948 the United States imported over 10,000 tons of high-grade mica, mostly from India and Brazil, valued at more than 15 million dollars; in the same period, domestic high-grade production totalled only 135 tons with a value of less than 50,000 dollars.

Since the war, world production of natural sheet mica has declined sharply, while production of ground mica has shown a continuing rise to meet the needs of roofing, paint, and rubber industries. The drop in sheet mica output reflects not only decreasing military demands for high-grade mica but also increasing labor costs which make hand-processing unprofitable. The usable sheet product is only a small fraction of the total processed volume. Some foreign countries can still process natural mica cheaply by hand methods, but producers in the United States are now concentrating on the production of "half-trim" mica.

Successful control of crystal orientation in the growth of synthetic mica would mean that machine methods could be used for large-scale domestic production of sheet mica. In this way mica synthesis could make the United States self-sufficient in high-grade mica insulating materials.

THE FRANKLIN INSTITUTE

STATED MONTHLY MEETING, MARCH 15, 1950

The Stated Meeting for the month of March convened in the Lecture Hall of The Franklin Institute Wednesday the fifteenth at eight-fifteen o'clock. There were about 225 people in attendance.

The President invited attention to the fact that the meeting was a bit early in the month and therefore members had not received their JOURNAL which contained the minutes of the previous meeting. He suggested that approval or correction of those minutes be made at the next meeting.

The membership of the Institute for the month of February was recorded by the Secretary as follows:

Sustaining	1
Active	49
Associate	43
Student	21

The total Institute membership as of the twenty-eighth of February was 5974.

The Secretary further announced the "Science is Fun" dinner which is to be given on March 30. This dinner will be given in honor of the gentlemen who have been members continuously of The Franklin Institute for fifty years or more. A preview of the Planetarium show "Rocket to Mars" will take place in conjunction with the dinner.

The President introduced the speaker of the evening, Dr. W. F. G. Swann, director of the Bartol Research Foundation, who spoke on the "Trends of Thinking in Modern Physics." Dr. Swann said modern physics has developed a trend in the direction of very abstract formulations of its theories. He discussed the attitude with which the student of natural philosophy must approach these matters in order to achieve understanding in a sense satisfactory to himself. He drew illustrations from classical theories of physics, the theory of relativity, and the quantum theory of atomic structure.

The meeting adjourned with a rising vote of thanks to the speaker.

HENRY B. ALLEN,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, March 8, 1950.)

HALL OF THE COMMITTEE,
PHILADELPHIA, MARCH 8, 1950.

MR. LEWIS P. TABOR in the Chair.

The following report was presented for final action:

No. 3166: Work of K. Hickman in the Field of Molecular Distillation.

This report recommended the award of a John Price Wetherill Medal to Kenneth Claude Devereux Hickman, of Rochester, New York, "In consideration of his ingenuity and leadership resulting in development of improved means in producing high vacua by the use of oil diffusion pumps; for significant advances resulting from the design of the centrifugal still and for his commercial adaptation of these improved practices to the recovery of Vitamins A and E from hitherto unusable raw materials."

JOHN FRAZER,
Secretary to Committee.

MUSEUM

How many times a day do you lift a telephone receiver, either to make or receive a call? Whether you do it only a few times a day or whether you are a constant user, the efficiency of the system is at such a high level that you will never pause to wonder how the whole process is possible. To reach for the telephone is almost as involuntary an action as to breathe. Only when one or the other action ceases to function smoothly do we ever begin to wonder about the processes involved.

The importance of the telephone in our civilization is immeasurable. The time it saves and the conveniences it provides cannot be calculated, but it stands to reason that this instrument which lies at our elbow and which brings the outer world within immediate reach is only the minute fragment of a complicated system. In fact, the system is so intensely complicated that the untechnical mind stands aghast before the contemplation of the multitude of details involved.

It is not enough to know that there are nearly forty million telephones in use in this country, nor that 177,000,000 calls are made daily. That knowledge fails to give us a grasp of how the daily miracle is performed. In order that visitors may be made acquainted with the elaborate organization of men and mechanisms which are combined to furnish the efficient service we have become accustomed to expect from the telephone system, an admirable exhibit is being installed in the Museum by the Bell System.

Space does not permit us to describe all the exhibits, since the display is so comprehensive in its provisions, but the visitor will obtain a clear picture of the care that is taken to transmit the human voice over space and the intricacy of the web of communication that makes it possible. Some of the exhibits will undoubtedly attract more attention than others, but we feel sure that the speech and hearing exhibit will exercise widespread fascination. In this the visitor is able to talk into a telephone and, after having spoken a message, to hear it as it will be heard by a listener at the other end of the line. From this exhibit, which epitomizes the process of telephony, the visitor can proceed to learn how a bewildering mass of dials, wires, relays, switchboards, and people are reduced to a well-ordered simplicity which enables us to talk across the street, across the continent, or across the ocean.

Naturally, a feature which will attract the majority of our visitors is the product of the research which is continuously in progress to improve the telephone facilities. One of the most notable of these products is the Transistor, a new type of electronic amplifier which differs from the familiar vacuum tube. The visitor will be permitted to learn how this tiny device will amplify the voice, and to see how effectively it performs the functions of a bulkier device formerly used for this purpose.

Another exhibit which should excite considerable interest illustrates a new method developed for growing the crystals used in carrying many conversations simultaneously over the same wires. Formerly, natural quartz crystals had to be imported from Brazil for this purpose, but the development for growing suitable synthetic crystals has rendered possible an expansion in supply as well as a release from dependence upon a foreign source.

These are merely a choice from the variety of exhibits which will be displayed and arranged to give visitors a better understanding of the system which knits together the majority of homes and places of business in this broad land. It is confidently felt that all those who see the exhibits will carry away a greater respect for the care and ingenuity lavished upon the maintenance of the system. A merely casual inspection will suffice to show that the seeming simplicity of making a telephone call brings into play scores of operations which spring promptly to our service when we reach out a hand and lift the telephone from its rest.

MEMBERSHIP

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS

March 15, 1950

ACTIVE FAMILY

W. W. Atterbury, Jr.
J. Walter Brand
James W. Clarke
William J. Crane
Herbert C. Doepken
William H. Shellington
George Voron

ACTIVE

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JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in the JOURNAL within the next few months:

MICHAELSON, HERBERT B.: Variations of Grid Contact Potential and Associated Grid Currents.

HERSHBERGER, W. D. AND L. E. NORTON: Servo Theory Applied to Frequency Stabilization with Spectral Lines.

HEREFORD, FRANK L.: Concerning Meson Produced Secondary Electrons Emerging from Thin Foils.

PENNER, S. S.: Application of Near-Equilibrium Criteria During Adiabatic Flow to Representative Systems.

HEPBURN, JOSEPH S.: The Influence of the Temperature and the Period of Keeping upon the Biochemical Changes in the Common Fowl *Gallus Domesticus*.

BEYER, ROBERT T. AND MING-YI WEI: An Extension of a Theory of Magnetic Amplifiers.

THORNTON, WILLIAM M., JR., AND EDWARD S. HAUBER: Zirconium as a Material for Fractional Weights.

WALLMAN, HENRY: An Electronic Integral-Transform Computer and the Practical Solution of Integral Equations.

LAUTERBACH, KENNETH E.: Specific-Surface Determinations of Uranium Dusts by Low-Temperature Adsorption of Ethane.

THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

The Airflow Slide Rule.[1]—LISBETH CROWELL.[2] Now that transonic and supersonic flight are a reality, engineers, scientists, and laymen are becoming increasingly interested in the phenomena of supersonic air flow. Although these phenomena can be understood best by realizing the magnitude of changes which occur in high-speed flow, even the most elementary supersonic-airflow calculations involve laborious computation. The functions relating airflow properties appear to be so formidable that few people bother to make numerical calculations. Graphs and tables which show these functions are available, but even these require some mathematical calculations before absolute values can be obtained.

Some automatic means should be available for computing airflow functions rapidly in order that available data may be used with maximum efficiency and to maximum advantage. The Airflow Slide Rule (see figures) was designed by the author to meet the need for such an automatic computing device. It affords an easy, accurate and rapid means of computing transonic and supersonic airflow relations.

Two types of relations are involved in airflow calculations: general and specific. The general relations are limited to the physical properties of air. The specific relations define the changes in airflow characteristics when the flow is isentropic, such as the theoretical flow from a wind tunnel reservoir, and when supersonic flow is changed by a normal shock wave, such as would occur in front of any blunt body in a supersonic air stream.

The flow properties or characteristics most commonly used in calculations are temperature (T), velocity (V), pressure (P), density (ρ), and Mach number (M). Mach number—the ratio of the velocity of flow to the velocity of sound in that flow—is the parameter, or single variable used in all high-speed airflow theory for relating the flow property changes which occur in either isentropic or normal shock-wave flow. Consequently, these relations of flow property changes are often referred to as the Mach Number Functions.

One face of the slide rule is labeled "Isentropic Airflow Relations." This face is devoted to the general and isentropic functions. The reverse face shows the normal shock functions plus a pitot-static pressure relation calibrated for both subsonic ($0.5 < M < 1.0$) and supersonic ($1.0 < M < 5.0$) velocity flow. Each theoretical relation is printed on the face of the Airflow Slide Rule after the calibrated scale used to compute that function. All calculation for flow, with Mach number within the range of 0.5 to 5.0, can be made with this computer.

Two sliding inserts are used with this device. One is calibrated for temperatures in degrees Kelvin (°K = °C + 273) and degrees Rankine (°R = °F + 460) and in velocities in feet per seconds (fps.) and miles per hour (mph.). The second is calibrated for pressures in pounds per square inch absolute (psia.) and densities in slugs per cu ft. The actual slug values are 10^{-4} times the printed values on the P, ρ slider.

The calibration of the Rule was developed from the general relations and the Mach-number functions. The general relations are:

$$(1)\ a = \sqrt{\gamma R T}, \qquad (2)\ V = aM, \qquad (3)\ \rho = 0.084\,\frac{P,\ \text{psia}}{T,\ °R},$$

where a = velocity of sound
γ = ratio of specific heats of air = 1.4
R = universal gas constant
T = free air temperature
V = Velocity
M = Mach number
ρ = density
P = static pressure.

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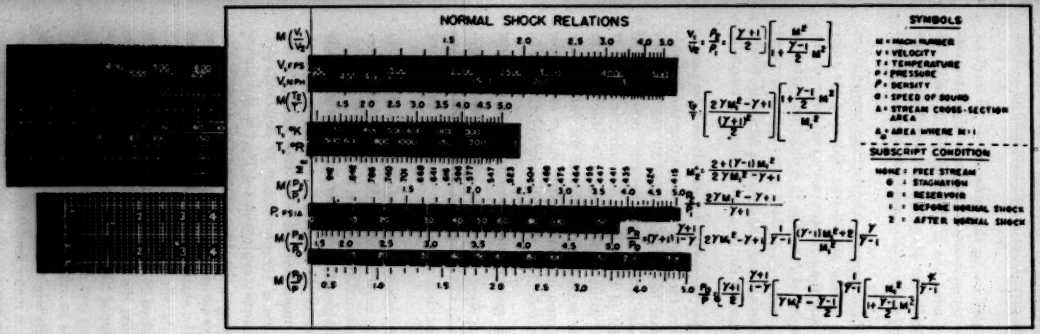
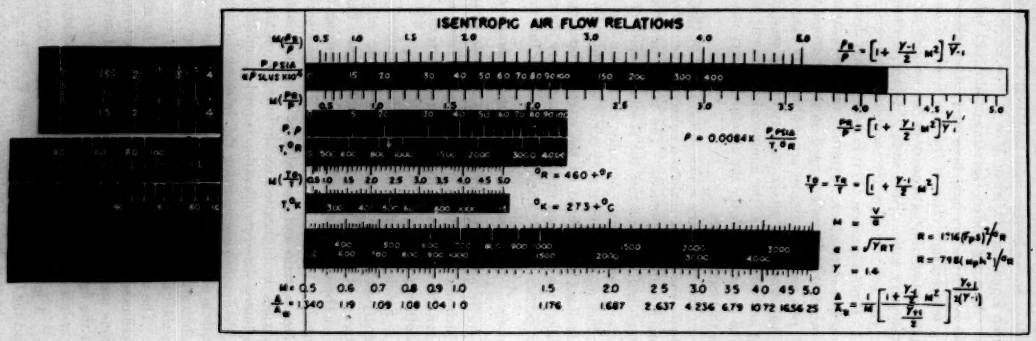
[1] Reprinted from *Aero Digest*, December, 1949.

[2] Research Engineer, The Franklin Institute Laboratories for Research and Development, Philadelphia 3, Pa.


NORMAL SHOCK RELATIONS
SYMBOLS
M = MACH NUMBER
V = VELOCITY
T = TEMPERATURE
P = PRESSURE
ρ = DENSITY
a = SPEED OF SOUND
A = STREAM CROSS-SECTION AREA
A* = AREA WHERE M = 1
SUBSCRIPT CONDITION
NONE = FREE STREAM
0 = STAGNATION
R = RESERVOIR
1 = BEFORE NORMAL SHOCK
2 = AFTER NORMAL SHOCK

FRONT


ISENTROPIC AIR FLOW RELATIONS
P PSIA
ρ SLUG/FT³
°R = 460 + °F
°K = 273 + °C
M = V/a
a = √γRT
R = 1716 (ft/sec)²/°R
R = 795 (mph)²/°R
γ = 1.4

BACK

The general relations are calibrated mainly by the orientation of particular scales on the slide rule. As can be seen from Equation (1), a is a fixed function of the free air temperature. Also, from Equation (2), when $M = 1$, $V = a$.

As noted above, the temperature and velocity scales are on the same slider. Therefore, the velocity scales have been so oriented relative to the temperature scales that, when the free-air temperature value of the flow is placed under the index line of the rule, the value of the speed sound always appears on the velocity scales opposite the $M = 1.0$ mark on the adjacent stationary scale. Also, since $V = aM$, a straight fixed scale of M values has been calibrated on the isentropic face for computing this relation.

From Equation (3) it is evident that density can be computed directly from temperature and pressure values. Pressure and temperature scales appear on different sliders of the rule. A window has been arranged on the isentropic face so that the adjacent calibrated edges of the P, psia and T °R. scales are visible. By placing the value of P on the P, ρ slider opposite the T value on the T slider, density can be read directly on the P, ρ slider opposite the arrowed 84 and 840 marks (the numerical equivalent of the constant in Equation (3)) on the T slider. The decimal point for the absolute density value in slugs per cu. ft. must be checked mentally in this calculation.

The stationary scales on each face of the slide rule, designated by M followed by parentheses enclosing a ratio as a subscript (i.e., $M_{(P_0/P)}$), are the scales for computing the variable Mach-number functions. The term "variable" is used here to designate all the functions employed in calculating the absolute values of particular temperature, velocity, pressure, and density conditions. Actually, each scale is a $\log_{10}$ plot of the ratio in the subscript with M values marked where they occur as a function of that ratio. All the variable Mach-number functions are printed on the edges of openings in the stationary section of the rule, so the sliders may be used with these stationary scales for computing the absolute values of the flow properties. Two "fixed" Mach-number functions also appear on the slide rule: A/A_* and M_2, where $\frac{A}{A_*}$ is the stream cross-sectional area expansion ratio required to change channel-flow velocity isentropically from sonic velocity to the free-stream speed. M_2 refers to the stream Mach-number after a normal shock. These are termed "fixed" functions since they always have the same value for a given free-stream Mach-number and are not used for evaluating another quantity.

Solving the fixed-Mach-number functions is merely a matter of reading the correct A/A_* or M_2 scale value opposite the desired M value on the adjacent M or M_1 scale. (M = free-stream Mach-number; M_1 = Mach-number before a normal shock wave.)

The operation of the Airflow Rule using the variable Mach-number functions is equally simple. Remembering that the M scales represent plots of the ratio in the subscript, and that, given a ratio and the value of the denominator (or numerator), it is necessary only to use straight multiplication (or division) to obtain the value of the numerator (or denominator), ordinary slide-rule operation is used for all the flow-quantity evaluations. That is, if a slider-scale number representing the value of the denominator in the ratio subscript of a particular scale is placed opposite the index line of the scale, the value of the numerator of the ratio in the subscript will always appear on the slider opposite the stationary Mach-number value used in the calculation.

The use of the *Airflow Slide Rule* and its simplification or ordinary airflow calculations can be illustrated best by two problems using every scale on the device. The symbols and notations used in these examples are printed on the Normal Shock Face of the rule, as shown in the figures.

EXAMPLE NO. 1

A jet plane was flying at about a 10,000-ft. altitude where the static pressure was 10 psia and the free air temperature −40° F. The pilot's Mach meter read 0.8. Assuming that these are the true flight conditions, what is the velocity of the aircraft, the speed of sound in, and the density of, the free air, and the total pressure and temperature (excluding radiation or internal cooling) experienced by the tip of the nose of the plane?

PROCEDURE

Since M is less than 1.0 no normal shock wave occurs. Therefore using only the *isentropic* face of rule,

1. Set $P = 10$ on the P slider scale, under the index on the left-hand side of this face.
2. Set $T = 420°$ F. $(460 - 40°$ F.) on the T slider scale under the index line.
3. Read:
   (*a*) Above the $M = 1.0$ on the M scale, $a = 1010$ fps., or 687 mph. on the V slider scales.
   (*b*) Above the $M = 0.8$ on the M scale, $V = 800$ fps., or 547 mph.
   (*c*) Opposite the $M = 0.8$ point on the $M_{(T_0/T)}$ scale, read: $T_0 = 470°$ R. or 260° K., the temperature of the nose of the plane.
   (*d*) Opposite the 840 mark on the T, °R. scale, read $10^3\rho = 209$ slugs (or opposite the 84 arrow on the T, °R. scale, read $10^4\rho = 20.9$) on the P, ρ slider scale. The density of the surrounding air is then .00209 slugs per cu. ft.
   (*e*) Above the $M = 0.8$ point on the $M_{(R_P/P)}$ scale, read $P_R = 15$ psia on the P, ρ slider. This is the pressure on the nose of the plane.

Calculation time for this example: three minutes.

EXAMPLE NO. 2

A supersonic blowdown-type wind tunnel operates with atmospheric intake and has a test section Mach Number of 3.1. Assuming that the atmospheric conditions are $R_R = 14.5$ psia, and $T_0 = 70°$ F. $= 530°$ R., find the:

(*a*) Area-expansion ratio from the tunnel throat required to produce this Mach Number with isentropic flow
(*b*) Density of the intake air
(*c*) Free-stream temperature, velocity, sonic velocity, and pressure in the test section assuming isentropic flow
(*d*) The pressure, P_0, which would be measured by a pitot probe in the test section
(*e*) The temperature, pressure, Mach Number and velocity which would exist in the stream immediately behind the normal shock wave caused by the nose of the pitot probe.

PROCEDURE

1. *Using the Isentropic Side of the Rule:*
   (*a*) Below the $M = 3.1$ point on the M scale, read $\frac{A}{A_*} = 4.7$ on the $\frac{A}{A_*}$ scale.
   (*b*) Set a value of $T_0 = 530°$ R. on the T, °R. slider scale directly below the 14.5 mark on the P, ρ scale. Above the 840 arrow on the T scale, read $10^4\rho R = 23.9$ or $\rho R = .00239$ slugs per cu. ft.
   (*c*) Set $P_R = 14.5$ above the 3.1 mark on the $M_{(P_R/P)}$ scale. Notice that the index line of the P scale is beyond the end of the P slider graduations or that P must be less than 1 psia. To avoid lengthening the slider size, values less than 1.0 psia have not been included; but these values can be obtained easily by using the pressure values as 1/10 of the value marked on the scale. Therefore, for this present calculation, put $P_R = 145$ above the $M = 3.1$ mark on the $M_{(P_R/P)}$ scale; and read P above the index mark of the $M_{(P_R/P)}$ scale as $1/10 \times 3.35$, or 0.335 psia.
   (*d*) Set $T_0 = 530°$ R. above the 3.1 marks of the $M_{(T_0/T)}$ scale, and opposite the index of the $M_{(T_0/T)}$ scale, read $T = 180°$ R. or 100° K.

Retaining the settings of the above two steps (1*c* and 1*d*),

(*e*) above the $M = 1.0$ line on the M scale, read $a = 662$ fps. or 450 mph. on the V slider.
(*f*) Above the $M = 3.1$ mark on the M scale, read $V = 2080$ fps. or 1390 mph. on the V scales.

2. With the same settings, but reversing the rule, *On the Normal Shock Side:*
   (*a*) Below the $M = 3.1$ point on the $M_{(T_2/T)}$ scale, read $T_2 = 282°$ K. or 509° R. on the T slider.

(*b*) Below the 3.1 point on the $M_{(P_2/P_1)}$ scale, read $P_2 = 1/10 \times 38 = 3.8$ psia. (It must be remembered, P is set at ten times its true value under the index line).

(*c*) Above the 3.1 mark on the $M_{(P_0/P)}$ scale, read $10\,P_0 = 43.8$ on the P slider, or $P_0 = 4.38$ psia.

(*d*) Above the 3.1 point on the $M_{(P_2/P_1)}$ scale, read the value $M = 0.469$ on the M_2 scale.

3. *Using the Normal Shock Side of the Rule*, set $V_1 = 2080$ fps. below the 3.1 mark of the $M_{(V_1/V_2)}$ scale. Below the index line of the $M_{(V_1/V_2)}$ scale, read $V_2 = 522$ fps. or 353 mph. on the respective V scales.

Calculation time for this example: eight minutes.

A comparison of the slide-rule results with computations made by a competent computing-machine operator using Mach Number Tables data showed that this early model of the slide rule resulted in third-place accuracies.

LIBRARY

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BOOK REVIEWS

THE MEANING OF RELATIVITY, by Albert Einstein. Third edition, including the generalized theory of gravitation. 150 pages, diagrams, 14 × 21 cm. Princeton, The Princeton University Press, 1950. Price, $2.50.

The *Meaning of Relativity* is the third edition of this work which was first published in 1922. In 1945 the second edition was published and it contained the significant advances in relativistic physics which came after 1922. Now appears a third edition and this contains in a second appendix a new theory which may well be the most significant paper in physics published in this century, outranking in magnitude both the restricted and the general theories of relativity.

The *Meaning of Relativity* has been reviewed too many times for this writer to attempt this. The feature of this edition is the second appendix which contains the Generalized Theory of Gravitation. In a brief paper, but 14 pages in length, Dr. Einstein has attempted to find an interrelation between electromagnetism and gravitation.

The problem of bringing these two major cosmic forces together in a unified field theory has absorbed the major energies of Dr. Einstein for the past 30 years. Though there have been numerous publications and many diverting influences in the life of Dr. Einstein we have been told that his major effort in the past three decades has been the goal of a unified field theory. In this theory Dr. Einstein has attempted to link the behavior of the electron, the proton, and the neutron with the forces of gravitation found in the macrocosmos. Of the Generalized Theory of Gravitation Dr. Einstein says, "It is a 'highly convincing' extension of the general theory of relativity, although, due to mathematical difficulties, I have not yet found a practicable way to confront the results of the theory with experimental evidence."

The aim of the new unified field theory seems to have been met in this latest attempt. At least the field theory is consistent in that both the gravitational field and the electromagnetic field—through Maxwell's equations—are represented. However, the concept to be truly all-embracing must bring the quantum theory into the field theory. At this time it is impossible to say for certain whether this has been achieved, though preliminary indications are favorable. Now it seems to be simply a question of time before the theory can be compared with the vast store of physical knowledge and the predictions which will ultimately come to test the theory.

Some physicists are quite skeptical of the new effort because nowhere does the Planckian constant "h" appear. At first blush it seems inconceivable that this "h" should not be a basic component of any theory purporting to yield the equations for elementary particles governed by quantum theory. However new concepts, especially those born of genius, are characterized by the boldness of their approach to problems and it may be that the "h" is somehow implied. Certainly the concept of a field due to the tiny particles of the microcosmos is not alien to the trend of modern thought. But whether the generalized theory of gravitation will yield equations governing this field is as yet uncertain. If the next few years should show this relation, then the revolution in physics begun at the turn of the century may be approaching completion.

I. M. LEVITT

LOUIS PASTEUR; FREE LANCE OF SCIENCE, by Rene J. Dubos. 418 pages, plates, 15 × 22 cm. Boston, Little, Brown & Co., 1950. Price, $5.00.

The name of Pasteur is known to everyone if for no other reason than that it appears on the cap of the morning milk bottle. Few men of science have had their lives recorded in such detail, but in spite of all that has been written there was still a niche to be filled and Dr. Dubos has filled it admirably with this book. No one could have brought higher technical qualifications to the task of evaluating the experimental and practical importance of Pasteur's contributions to science than the author, who was born and educated in France and is a microbiologist of repute. But Dr. Dubos has brought more than technical qualities to the fulfillment

of his task. He has a gift of writing which makes him a rare expositor to the non-technical reader.

Louis Pasteur came from a humble provincial home but he planned his career so that he might devote himself to lofty studies of theoretical science. Circumstances ordained that he would often have to forsake the experimental for the practical, but in everything he undertook he never lost sight of the urge to create. With a slavish devotion to his work he attained success through his perception of "the infinitely great power of the infinitely small." On reading his scientific papers one gathers that he was swept forward with an inner compelling logic by a process of natural evolution from the physiochemical, through the chemical and biochemical, to the purely biological point of view. Dr. Dubos makes clear, however, that the different aspects of the work did not stem from each other. He points out, too, that Pasteur's great discoveries sprang from intuitive visions and were followed by periods of intense investigation which provided the necessary experimental evidence.

Pasteur was an extraordinary man, even for a scientist. He labored in solitude in libraries and laboratories, he shared in the practical problems of the shop and the fields, and he fought when necessary from the tribunes of academies, in the press, and before the public. His was a strenuous life, but Dr. Dubos contrives not only to present an adequate picture of the man and his work but also to fit it deftly into the historical background. We see the young Pasteur grow in intellectual stature as he progresses from studies in molecular structure, in the physiological mechanisms of fermentation, and in the pathogenesis of infectious diseases. Behind all these labors we perceive his passionate hope that he was aiding in the restoration of his country to the glory of leadership in science which had been forfeited in the first half of the nineteenth century.

By the time the Pasteur Institute was founded—a great international tribute to the scientist—his work was accomplished, yet he continued to haunt the laboratories and to inspire his disciples. "He was the symbol of the great creators who, despite poverty and at the cost of sacrifices and suffering, establish the foundations of science that less gifted men may add slowly to the great structure arising from the struggles of genius."

The book is more than a biography of a great scientist. It offers an adequate history of microbiology and presents some glimpses of the philosophy of science as caught by a penetrating eye and drawn with a shrewd hand. It is one of the best studies of the life and work of a scientist this reviewer has read for many a long day.

T. C.

COSMIC RADIATION AND ITS BIOLOGICAL EFFECTS, by Victor F. Hess and Jakob Eugster. Second edition, 178 pages, drawings, photographs, maps, 16 × 23 cm. New York, Fordham University Press, 1949.

The first section of this book (83 pages) was written by Dr. Hess and comprises an excellent summary of the physics of cosmic radiation. Dr. Hess, who was awarded the Nobel Prize in Physics in 1936 in recognition of his work in cosmic radiation, has written this section in simple, lucid style and with a minimum of purely technical detail. The scientist in other fields, or the intelligent layman, can obtain a satisfying insight into one of the most interesting fields of modern physics.

The second section of this book was written by Dr. Eugster and was translated from the original German by Dr. Richard Pope and Dr. E. F. Hartree. It consists, in addition to a review of the literature, of a description of various exploratory experiments which were performed by the author. Although the techniques are fully described, the detailed results of the experiments are seldom given. Consequently, it is difficult to judge whether the results of the experiments are properly evaluated by the author. Dr. Eugster tentatively concludes that cosmic radiation caused the inhibition of bacterial growth, the death of bacterial spores, and inhibited the germination of certain plant seeds (*Avena sativa* and *Phleum pratense*). However, the seeds of *Linum usitatissimum*, after exposure to radiation are described as showing more rapid germination than did the control seeds, although the subsequent development of the plants was inhibited. Dr. Eugster tentatively concludes that reproduction of *Drosophila*

melanogaster is stimulated by radiation, although reproduction in rabbits is inhibited. He also has calculated that the incidence of spontaneous carcinoma in mice was four times higher in animals exposed to radiation than in the controls. It is indeed unfortunate that the author felt it necessary to make a cursory investigation on such a variety of experimental material. It would have been much more valuable if fewer subjects had been thoroughly investigated by truly quantitative methods, and the results presented in detail. As the matter stands, the reader can come to no definite conclusion as to the biological effect of cosmic radiation on the basis of the work presented here.

LOUIS DESPAIN SMITH

MEASURING THE UNIVERSE, by Oliver Justin Lee. 170 pages, illustrations, plates, 15 × 22 cm. New York, The Ronald Press Co., 1949. Price, $3.00.

In physical size man stands approximately halfway between the smallest unit in the microcosmos and the largest unit in the macrocosmos. Man's determination of this position in the universe results from his studies and researches centered on the fascinating problem of distance determination. With almost mystical insight the scientist has projected his mind through the length and breadth of the universe to probe and search for the answer to our position in the scheme of things.

Dr. Lee has written a fascinating book on the determination of the distances by the astronomer. He has shown how the thinking of the scientist has been influenced by the distance concept and how this concept has led to an understanding of our surroundings. Even the terms used in arriving at distances have their own special flavor. Dr. Lee has presented the story in such a fashion as to give the reader a sense of admiration for those pioneers in the sciences who dedicated their lives to the problem of distances.

The determination of the prime distances have been so well explained and carefully written that the book may well serve as a source book for those interested in how the most important distances to man have been established. The book contains a wonderful exposition of the derivation of the astronomical unit—the basis for almost all distance determinations. Dr. Lee shows how the distance to the sun has been derived with so much precision that, "The percentage of accuracy is about the same as if a man who is six feet tall would give his height to 0.007 of an inch, which is about twice the thickness of ordinary writing paper."

The determination of the distances to the stars by the various methods known to the astronomer is presented in as brilliant and lucid a piece of writing as this reviewer has encountered. The determination of distances to such remote objects as stars has taxed the ingenuity of the astronomer to the utmost. In this section of the book Dr. Lee shows how, having determined the distances to the nearer stars by the ordinary trigonometric method, the astronomer has built on this a structure which has permitted him to plumb the very depths of space.

The various relations between proper motions and parallax, spectral characteristics and parallax, and the period and brightness of the cepheid variables have been thoroughly examined to show how they yield the distances to the stars.

There is one section of the book which is implied but omitted and that is the uncovering of the distances involved in the atom. It is regrettable that this has been omitted because it is one of the most interesting chapters in contemporary physics. The determination of the sizes of the atom through the use of crystals, the Rutherford scattering experiment to determine the size of the nucleus of the atom and the highly interesting method used to derive the radius of the electron are parts of the story which should have been told. The uncovering of these minute distances is to the reviewer's mind one of the most interesting and romantic in our scientific literature.

The omission of the above does not detract from the sparkling style and humor in which the book abounds. *Measuring the Universe* is highly recommended.

I. M. LEVITT

ORGANIC SYNTHESES, VOL. 29, edited by C. S. Hamilton. 119 pages, illustrations, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1949. Price, $2.50.

Volume 29 of "Organic Syntheses," under the editorship of C. S. Hamilton, contains detailed instructions for the syntheses of 34 compounds; 1-Acetylcyclohexene, Acrylic acid, *m*-Aminobenzaldehyde Dimethylacetal, 5-Amino-2,3-dihydro-1,4-phthalazinedione, beta-Benzoylacrylic Acid, alpha-Bromoheptaldehyde, tert.-Butylamine and tert.-Butylamine Hydrochloride, Raney Nickel Catalyst, 4-Chlorobutyl Benzoate, 2-Chloromethylthiophene, gamma-Chloropropyl Acetate, 2,6-Dichlorophenol, alpha, alpha-Diphenylacetone, Ethyl beta-Anilinocrotonate, Ethyl 2-Pyridylacetate, 1-Ethynylcyclohexanol, Homophthalic Acid and Anhydride, beta-(2 Hydroxyethyl-mercapto) propionitrile, Indazole, Isoprene Cyclic Sulfone, Methacrylamide, *m*-Methoxybenzaldehyde, 1-Methylaminoanthraquinone, 1-Methylamino, 4-bromo anthraquinone, 2-Methyl 4-Hydroxyquinoline, *m*-Nitrobenzaldehyde Dimethylacetal, *p*-Nitrobenzonitrile, 5-Nitro 2,3-dihydro 1,4-phthalazinedione, Oleyl Alcohol, alpha-Phenylcinnamonitrile, Protocatechuic Acid, 2-Thiophene aldehyde, 1,2,5-Trihydroxypentane, and Trimethylene Oxide.

Of particular interest in this issue is the preparation of W-6 Raney Nickel Catalyst which is especially useful in low temperature and low pressure hydrogenations. Compounds containing aldehyde, ketone, oxime, and nitro groups as well as benzenoid and pyridinoid nuclei lend themselves particularly well to hydrogenation with this catalyst.

The organization of material in this volume is the same as that which characterizes the previous volumes; that is, the experiments have been prepared and checked carefully as to optimum conditions and yields. At the end of each synthesis, other methods of preparation and notes covering steps in technique not covered in the procedure, are included. A cumulative index comprising material from Volumes 20 through 29 is inserted for reference purposes.

"Organic Syntheses" is well recognized as one of the most useful tools of the organic chemist. It provides a source of information for particular compounds and also for special synthetic methods applicable to many problems. No scientific library should be without every volume of this extremely valuable series.

DONALD H. RUSSELL

THE PROTECTION OF TRANSMISSION SYSTEMS AGAINST LIGHTNING, by Walter W. Lewis. 418 pages, drawings and photographs, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1950. Price, $8.00.

"Transmission Systems" in this book refers to commercial electric power distribution systems. As one of a series on the subject of electrical engineering this book is another written for the General Electric Advanced Engineering Program.

Besides being an engineering work of importance to the profession this is a fascinating book which contains some enjoyable reading. This is partly because lightning is a glamorous subject in which some of Nature's most gigantic and spectacular phenomena are involved, and partly because of the body of measured facts reported by an author who has lived with his subject for 25 years and has reduced it from a speculative topic to a science.

Beginning with an explanation of several theories pertaining to the accumulation of electricity in clouds the author proceeds to an impressive documentation of the subject by presenting the results of extensive measurements of cloud-field gradients by ground and airborne investigators. Similarly, the entire volume is heavily documented with the findings of many investigators working in scattered places over a long period of time. The mechanism of the lightning stroke, its composition, intensity, velocity of propagation are described in detail under various long-term study projects. Of interest also are the many ingenious instruments and techniques employed to obtain the data.

It appears to be a seldom realized fact that many of the first large transmission systems had to be erected with more daring than knowledge of consequences. Accounts of great economic and equipment losses suffered due to ignorance of principles to be applied in lightning protection hint at the price that was paid for much of our knowledge prior to the establishment of present-day high-voltage research facilities.

The later portions of the book are devoted to the details of modern practice in which a careful balance is maintained between line and equipment insulation. The communications engineer may also find it helpful to refer to this book in providing protection for modern line-of-sight installations which are notably situated in vulnerable locations whose height and lack of other obstructions are their greatest virtue.

C. W. HARGENS

MACHINE TOOLS FOR ENGINEERS, by Charles R. Hine. 355 pages, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Co., 1950. Price, $3.50.

The author of *Machine Tools for Engineers* easily accomplishes his purpose for writing the book: to introduce the student to the fundamentals of machine tools and production processes. Amply illustrated with appropriate photographs, drawings and charts, the well written text, describing cutting tools, measuring instruments, abrasives and grinding, and such various machine tools as the lathe, shaper, planer and boring machines, is an excellent reference not only for the student but for any engineer. Since the college course from which the book evolved was based upon many conferences with industrial leaders and plant visits, the book contains much practical information.

Following an introductory chapter discussing the engineer and the machine tool, cutting tools and measurement are described in separate chapters. The use, purpose and description of all such measuring instruments as calipers, dividers, micrometers, gage blocks, feeler gages, optical flats, electric gages and air gages are given. Subsequent chapters are devoted to discussions of the function, purpose and use, principal parts, types, and, in some cases, short histories of the lathe, the drill press, the shaper, the planer, the milling machine, the boring machine and the turret lathe. Other chapters are concerned with threads and threading, broaching, abrasives and grinding, microfinishes, gear making, jig boring and sawing. As mentioned, the many fine photographs and drawings add greatly to the interest and value of the volume.

Each chapter contains pertinent references for continued study of the particular subject, and the appendix has a list of excellent books and periodicals on the general phase of manufacturing processes.

E. W. HAMMER, JR.

A GENERAL KINETIC THEORY OF LIQUIDS, by M. Born and H. S. Green. 98 pages, 18 × 26 cm. Cambridge, The University Press, 1949. Price, $2.25.

This book is a collection of six papers originally published in the *Proceedings of the Royal Society* during the period 1946–47. They represent a brilliant initial mathematical attempt to provide a kinetic treatment of the liquid phase through the medium of general classical theory and quantum mechanics.

The first paper shows how distribution functions for a set of particles may be obtained, after preliminary definitions, by considering the probability relations between position and velocity, and the general equations of motion. The Maxwell-Boltzmann equations for gases then follow as a special case. The second paper demonstrates, independently of statistical mechanics, the implicit equilibrium properties of the equations of the first paper, involving consideration of Boltzmann's H (his counterpart of entropy) theorem. The paper concludes with a further consideration of equilibrium and thermodynamic properties, and the separation of the mathematical material into the liquid and gaseous phases.

The third paper, by considering hydrodynamics, intermolecular forces and thermal motion, extends the theory to dynamical properties and yields expressions for the viscosity and the thermal conductivity of liquids.

Finally, the fourth and sixth papers are concerned with the translation of the classical treatment of the previous papers into that of quantum mechanics using matrix formulation. These revised equations are then used in an attempt to explain some of the unusual ("super") properties of liquid helium II.

The fifth paper is a derivation of thermodynamic properties on the basis of kinetic theory with a quantum mechanical treatment.

Since the papers are reprinted verbatim, much more is left to the reader's knowledge and ingenuity, of course, than would be in the usual treatise or text book. However, these most important contributions to the theory of liquids are, to the interested reader, well worth the extra interpretive effort required, and worthy of a great quantum mechanical pioneer and his colleague.

ALBERT F. MYERSON

INTRODUCTION TO THE THEORY OF PROBABILITY AND STATISTICS, by Niels Arley and K. Rander Buch. 236 pages, diagrams, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1950. Price, $4.00.

This book presents the theory of probability and statistics in a fashion ideally suited for use by students of the sciences and engineering. The mathematical background required for a profitable reading of this book is only a knowledge of the calculus. The first three chapters discuss the fundamental concepts and the theorems underlying probability theory in a lucid and concise manner. Further chapters discuss, with both rigor and clarity, random variables, distribution function, dispersion, mean values and limit theorems. The theory of statistics as presented follows the school of R. A. Fisher. The chapters on the relation of probability theory to the theory of errors and to the theory of adjustment are of particular value to those whose professions require the analysis of data. Conciseness in treatment has been obtained by developing the theory of adjustment by means of matrices. For the uninitiated, the appendix contains an adequate discussion of matrix methods. Throughout the book there is a large collection of both illustrative examples and problems of theoretical and practical interest. In summary, this text is an excellent combination of the rigorous approach to probability theory, together with the practical applications.

EZRA KRENDEL

DEVELOPMENT OF AIRCRAFT ENGINES, by Robert Schlaifer; and DEVELOPMENT OF AVIATION FUELS, by S. D. Heron. 784 pages, 14 × 22 cm. Boston, Harvard University, 1950. Price, $5.75.

If the reader is looking for a technical treatise on the details of aircraft engine design and on the chemical and thermodynamic properties of aviation fuels, he will be disappointed in this book.

If, however, he is interested in the history of the development of these engines and fuels, in the parts which government and industry played therein, their proportional responsibilities in the successes and failures, Messrs. Schlaifer and Heron tell the story in a detailed and yet most interesting manner. It is apparent that this book is the result of a tremendous amount of research and study of a type that could only be done by persons who had been deeply involved in this development and had lived with a large part of it.

Interspersed with the history are the authors' personal feelings and conclusions. They feel that direct government development of aircraft engines has proved futile; that governmental control has slowed down development; but they agree that without government sponsorship and direction, much less would have been accomplished,—a sort of necessary evil.

One may even find ammunition for the case against unification of the air services in the stimulation which resulted from the rival sponsorships of the Army and Navy air arms, and in the check which one had on the other.

It is brought out that while in Great Britain, developmental costs were almost wholly government sponsored, and in this country largely done directly at private risk, the results have been approximately equal and no good case can be made for either system.

Whether the conclusions drawn and solutions recommended are valid, the reader must decide for himself. The facts upon which they have been based are abundant and well documented, and the qualifications of the authors are unquestioned.

This book is a valuable addition to the documentation of the history of American aviation —a sort of H. G. Wells "Outline of History" for this particular field.

R. S. BARNABY

MATRIX ANALYSIS OF ELECTRIC NETWORKS, by P. Le Corbeiller. 108 pages, diagrams, 14 × 22 cm. Cambridge, Harvard University Press; New York, John Wiley & Sons, Inc.; 1950. Price, $3.00.

The 100-page monograph on matrix mathematics applied to electrical circuits treats numerous problems of practical importance. These are primarily associated with communications rather than power engineering, although the techniques for solving transformation problems will be useful in the latter. For completeness the author introduces the reader to the subject by a review of matrix notation, theory and the details of simple and more complicated manipulations.

Although the book commences with preliminary concepts of the matrix product exemplified by cascade connection of four-terminal networks, more advanced ideas pertaining to the node-pair method applied to general vacuum tube couplings are presented later. Both a-c. and d-c. problems are considered. The mesh method of Gabriel Kron is highlighted after an explanation of its relation to the familiar Kirchhoff and Maxwell mesh methods. Problems involving asymmetric coupling in which coupling impedances are unequal, as exemplified by triodes in which the grid currents are not negligible, are handled with facility by the Kron method. Although these particular examples are given, the mathematics is derived in sufficiently general form that it may easily be applied to other problems.

A working knowledge of the principles illustrated in this monograph would undoubtedly enable an engineer to use matrix methods with greater facility. The volume is considered by its author to be introductory to Kron's "Tensor Analysis of Networks."

C. W. HARGENS

ELECTRONIC ENGINEERING MASTER INDEX FOR 1947–1948. 339 pages, 17 × 25 cm. New York, Electronics Research Publishing Co., 1950. Price, $19.50.

Have you had occasion to use the previous issues of this book? If so, you will be glad to know that 1947–1948 *Index* is now available. If you want a comprehensive bibliography on any phase of electronic engineering, this book puts it at your finger tips.

Every technical library should have a copy on its reference shelf. Electronics engineers will want a copy of their own especially if a library copy is not easily accessible. And any other engineer or technician interested in the application of electronics to his work or the way in which electronics uses his product will find the subject indexed alphabetically. How are plastics used? Do you have a process control problem? How is electronics applied to medicine? These subjects plus all of the expected radio, television, circuit and tube theory and practice are indexed.

Material for the *Index* is drawn from approximately 250 major international scientific magazines and journals published between January 1947 and January 1949. The listing of declassified U. S., British, and Canadian wartime documents and the listing of electronic patents extends the field covered by the index. To complete the indexing of technical material, engineering books published in the U. S. during 1947 and 1948 as well as some foreign books are classified under the proper subject heading and listed in a separate bibliography.

To tie in this book with the previous editions, articles written since 1925 and listed in the *Electronics Engineering Master Index* are cross indexed.

With the large number of articles published yearly in the field of electronics this book fulfills a very definite need for a comprehensive bibliography of electronic subjects.

E. A. MECHLER

EARTH WAVES, by I. Don Leet. 122 pages, illustrations, 14 × 22 cm. Cambridge, Harvard University Press, 1950. Price, $3.00.

This short Harvard monograph in Applied Science summarizes the available knowledge on earth waves. In places the material is scanty, but, as emphasized by the author, this is due to a lack of quantitative, observational data. In spite of this lack, it seems in several instances the material could have been presented more clearly.

The measurement, types, and transmission of earth waves are discussed in separate chapters. Since all earth waves are recorded and measured by seismographs, the theory of the

instrument is detailed rather completely. Such subjects as simple harmonic motion and forced vibrations are developed from elementary principles to illustrate the functions of the seismograph. The three essential parts of the seismograph—the inertia system, the coupling system and the recording system—are described. In the second chapter each of the several types of earth waves, the body waves consisting of longitudinal *P*-waves, transverse *S*-waves, and the surface waves consisting of Rayleigh *R*-waves, Love *Q*-waves, hydrodynamic *H*-waves and coupled *C*-waves are discussed.

In the third chapter, the transmission of the various types of waves through the earth is described. The practical use of artificially induced earth waves for determining geological structure and oil deposits by means of a portable seismograph is also explained with the aid of several examples. The short, final chapter describes microseisms, small waves, apparently caused by such meteorological disturbances as hurricanes, typhoons and cyclones originating over large bodies of water, and how they have been utilized to locate such storm centers.

The volume is enhanced by numerous illustrations.

E. W. HAMMER, JR.

MATERIALS ENGINEERING OF METAL PRODUCTS, by Norman E. Woldman. 583 pages, illustrations, 16 × 24 cm. New York, Reinhold Publishing Corp., 1949. Price, $10.00.

The author of this book, who is a faculty member of the Cooper Union School of Engineering and of the Stevens Institute of Technology, addresses himself to the professional engineer in the metal-working industries and sets himself the double task of introducing the reader to "materials engineering" and of presenting a large amount of pertinent information about materials (commercially available in the U. S. A.) required for the rational solution of materials-engineering problems relating to several broad classes of metal products.

From the point of view of a "materials engineer," the "design" of a metal product is determined not only by considerations of the required behavior of the finished product under specific service conditions, but also by considerations bearing on the consecutive stages of fabrication and processing during its manufacture.

The subject matter is sub-divided according to the several principal types of metal products for which different classes of materials are used, namely: (1) light-weight construction (Al- and Mg-alloys, high-strength—low-alloy and stainless steels; (2) mechanical products such as gears (steels, cast iron, bronzes, non-metals), springs (C-, Cr-V, Si-Mn, and stainless steels, Cu- and Ni-alloys), bearings (babbits, Cu-Pb-alloys, bronzes, Al-base alloys, non-metals), and threaded fasteners (C-, alloy- and stainless steels, Cu-, Al- and Ni-alloys); (3) Electrical products such as magnets, contacts, bimetal thermostats and resistors.

The fourth section deals with the theoretical and practical aspects of corrosion and corrosion-resistant metals (Ni-, Ni-Mo-Fe- and Ni-Cr-Mo-Fe alloys, stainless steels, Al-bronzes), as well as with metals for high-temperature service (alloy steels, age-hardening, alloys, heat-resistant alloys).

A section of fifty pages is devoted to methods of mechanical and non-destructive testing and the commercially available testing equipment.

Within each section, a discussion of the typical uses, service requirements and principles of material selection is followed by a detailed presentation of the composition and pertinent properties of the materials used in the type of product considered. The behavior of these materials during mechanical working, heat treatment, carburizing, nitriding, machining, welding, and various other types of processing or fabrication, as the case may be, is then presented. Lastly, the problem of design on the basis of available information about the materials is discussed at length.

Throughout the text, and in an Appendix of 46 pages, very extensive data, furnished in many instances by the producer of the material are given on the nominal composition, the mechanical and pertinent physical properties, the effect of temperature, mechanical working or heat treatment, etc. for a large number of metallic and some non-metallic materials. This feature renders this volume a convenient reference book for the designer and production engineer.

In all, this book will serve well as an introduction to the art of "materials engineering"

and will be of great assistance to those readers who face the problem of selecting a material best suited for their product.

This reviewer feels that the general utility of the book would be increased if a glossary of the engineering terms used throughout the text would be compiled in an Appendix for ready reference.

O. R. SPIES

BOOK NOTES

SOILS, THEIR PHYSICS AND CHEMISTRY, by A. N. Puri. 550 pages, illustrations, 15 × 23 cm. New York, Reinhold Publishing Corp., 1949. Price, $7.00.

Dr. Puri offers in this volume a factual presentation of soil data in which he has endeavored to show the possibility of arriving at significant generalizations. Intended for the beginner, the book tries to avoid controversial theories by using the author's own extensive work as a basis.

Beginning with the simple principle of acid-base equilibrium, the author propounds the concept of soil, freed from extraneous matter, as an acid and reaches the conclusion that natural soils are merely mixtures of saloids of the soil acidoid.

The volume is divided into three main sections, each of which is developed at length—chemistry of the soils, mechanical analysis of the soils, and soil moisture. The work should be of interest to students of soils and their yields.

ENGINEERING MECHANICS, by Archie Higdon and William B. Stiles, in collaboration with Arthur W. Davis and Herbert O. Ustrud. 505 pages, diagrams, 15 × 23 cm. New York, Prentice-Hall, Inc., 1949. Price, $6.65.

This new textbook on engineering mechanics emphasizes the free-body diagram technique which the authors feel contribute much toward accuracy of visualization, clarity of thought, and ease of analysis. For equations of relative motion, the algebraic solution has been stressed instead of the graphic solution as placing more emphasis on fundamentals.

The text is designed for a two semester course, although it may be adapted to one term by omissions, several of which have been suggested. More than 1100 problems provide ample variety to permit assigning different problems from year to year. Instructors of engineering mechanics will undoubtedly wish to consider this work for possible adoption.

LEXIQUE TECHNIQUE, FRANCAIS-ANGLAIS ET ANGLAIS-FRANCAIS CONCERNANT LE MATERIEL DE TRAVAUX PUBLICS. 176 pages, 13½ × 21 cm. Paris, Institut Technique du Batiment et des Travaux Publics, 1949. Price, 700 francs plus postage.

To meet a need felt by French contractors in securing plant and spare parts from American or British sources, and in operating and maintaining such equipment, this dictionary has been prepared. The work covers engineering equipment in most of its various phases including civil, electrical, mechanical, and metallurgical. The English-French section is longer than the French-English section. Where English and American nomenclature differ, an effort has been made to indicate both, although only "tyre" is given and not the more familiar, to us, "tire."

To further facilitate its use as a trade tool, extensive conversion tables from British to metric units and *vice versa* have been included. Several typographical errors have crept in, most of which are corrected in an errata list. The dictionary should serve a useful purpose in fostering international trade for it includes words and expressions not generally available elsewhere.

RADIO OPERATOR'S LICENSE Q AND A MANUAL, by Milton Kaufman. 608 pages, illustrations, diagrams, 13½ × 22 cm. New York, John F. Rider Publisher, Inc., 1949. Price, $6.00.

The present volume is one of the question and answer type which have proven popular for the prospective candidate for a radio operator's license. Based on the latest government publications and claiming up-to-dateness as of the time of publication, this volume offers more than just answers to questions. With most questions there is included a discussion which amplifies the reader's understanding of the question. There are frequent cross references to allied questions, numerous diagrams and an index to the technical material.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

***Abstract of* Alkaline Phosphatase Activity and Desoxyribonucleic Acid.**—M. H. ROSS AND J. O. ELY (*J. Cellular Comp. Physiol.*, **34**: 71, 1949). Alkaline phosphatase shows very slow action on highly polymerized desoxyribonucleic acid. The effect becomes progressively greater as the molecular weight is reduced, presumably due to attack of the enzyme on terminal phosphate groups which are more numerous in the smaller molecular weight samples. When the concentration of highly polymerized desoxyribonucleic acid is adjusted to give equivalent terminal phosphate concentration as in a depolymerized sample, the enzyme action is essentially the same.

The sodium desoxyribonucleate preparations of various molecular weights were obtained by treatment with X-rays, heat, H_2O_2, sonic vibration, HCl and desoxyribonucleic acid depolymerase. The enzymic activity in fixed tissues of rat intestine was found to occur with very dilute substrates, first in villi cell nuclei only, then in villi cytoplasm, and then in the other duodenal structures. The same order of appearance of activity was found with adenylic acid and with sodium glycerophosphate when the substrate concentration was adjusted to the same number of molecules. Phosphatase activity which had been suppressed by sodium arsenate returned after re-incubation in an arsenate-free medium but when suppressed by cadmium nitrate did not return after re-incubation in a cadmium-free medium.

The phosphomonoesterase found in fixed intestinal tissue which hydrolyzes the terminal phosphoric acid groups from desoxyribonucleic acid is probably the same one which attacks the phosphoric acid group of adenylic acid and sodium glycerophosphate.

***Abstract of* Electrophoretic and Serological Properties of the Non-Dialyzable Growth Products of *Vibrio cholerae*.**—LAURA E. KREJCI, LUCILE SWEENY, AND ROBERT K. JENNINGS (*Arch. Biochem.*, **24**: 55, 1949). The intact growth products from several strains of *V. cholerae* were shown by electrophoresis to contain three main constituents, A, X, and B, each of which usually separated into two or more sub-constituents. The polysaccharides isolated from the growth products contained the A and B constituents also, but in different proportions; the X constituents were missing. In both the intact growth products and the polysaccharides there was evidence for reversibly dissociable interaction of the sub-constituents.

The X and B constituents of the intact growth products possessed antigenic activity. The X constituent produced inflammation at the site of injection. The heat-labile antigens appeared to be associated with the X constituents, the heat-stable O-group 1 antigen with the B constituents.

***Abstract of* Neutron and Roentgen-Ray Effects on Protein Content of Rat Intestine.**—M. H. ROSS AND J. O. ELY (*Am. J. Roentgenol. Radium Therapy*, **62**: 718, 1949). A reduction in the amounts of tyrosine, tryptophane, arginine and nitrogen, which are important constituents of protein, is evidence of a reduction in the amount of protein after neutron and roentgen irradiation.

Cytochemical determinations of the total protein content in tissue sections was estimated through the study of the tyrosine and tryptophane ultraviolet absorption at the wavelength 3654 Å. The tyrosine and tryptophane content was markedly decreased in the crypts of Lieberkühn 24 hr. after 600 r total body roentgen irradiation, but there was no significant change in the tyrosine and tryptophane content of the crypts of Lieberkühn 24 hr. after 56 n total body neutron irradiation. A considerable decrease in the tyrosine and tryptophane content of the crypts of Lieberkühn was found 24 hr. after 100 n or 200 n total body irradiation respectively.

The arginine content was found to decrease 24 hr. after 600 r or 200 n total body irradiation.

The percentage of nitrogen was not markedly changed in the acetone dried intestinal mucosa in irradiated or non-irradiated fasted animals. The amount of acetone-dried intestinal mucosa was significantly changed after 24 and 48 hr. in those animals irradiated with 200 n, but was not changed in those non-irradiated animals whose food had been restricted for the same periods of time. The calculated total nitrogen for the acetone-dried intestinal mucosa was found to be greatly decreased 24 and 48 hr. respectively after 200 n neutron radiation, but there was no significant reduction in non-irradiated fasted animals.

***Abstract of* Neutron Effects on Alkaline Phosphatase of Rat Intestine.**—M. H. ROSS AND J. O. ELY (*Am. J. Roentgenol. Radium Therapy*, **62**: 723, 1949). The effect of neutron irradiation on the alkaline phosphatase of the rat duodenum was investigated. An increase in alkaline phosphatase activity at pH 9.0 was found in the nuclei and nucleoli in the cells of the crypts of Lieberkühn when beta-glycerophosphate, desoxyribonucleic acid, ribonucleic acid, adenylic acid, guanylic acid, or diphenyl phosphate was used as the source of organic phosphorus.

No difference could be observed between the phosphatase activity of the cells in the villi of the duodenum from irradiated and from non-irradiated animals.

No difference in phosphatase activity at pH 7.0 was noted in any of the parts of the duodenum of irradiated and non-irradiated animals with any of these substrates.

CURRENT TOPICS

Shot Blast Cleaning Machine.—Through the application of a special process for the internal cleaning of engine blocks, Ford Motor Company has practically ruled out the possibility of engine trouble developing due to inadequate block cleaning, Logan Miller, general manager, Rouge Division, said.

The process employs a new shot blast cleaning machine that is the first of its kind in the automotive industry. It guarantees additional protection against core sand remaining in the internal cylinder block water passages.

The new pressure blast machine supplements conventional equipment now in use to remove burnt core and molding sand and scale from interior and exterior surfaces of the motor block casting.

Ford ultimately plans to equip each of the production foundry's motor block cleaning systems with the new compressed air blast cabinets.

The installation provides the latest known means of insuring removal of core sand and foreign material left inside the block following casting.

Possibility of obstructions developing in water circulation, thereby creating faulty or broken cylinder blocks in cars during operation is minimized.

The new blasting technique works this way: Motor blocks to be blasted are brought by conveyor one at a time into the blast cabinet. Two work positioning arms, one for each side of the casting, register the block in correct position for applying the blast of shot. The blast is blown through sixteen nozzles under air pressure of 85 pounds per square inch.

The air-driven shot strikes hidden sand or scale, freeing it from the metal's surface. The loosened material then can be removed from the block's interior.

Surfaces of the water jacket, crevices and thin sectional openings are blasted clean—exposing bright virgin metal to the cooling water circulated from the radiator.

The material used in the blast process consists of iron or steel shot and grit. The air blast equipment operates automatically, the blast period being controlled by an automatic timer.

As a result of this efficient internal cleaning, water passages in the block are smooth and free from flow interferences and result in cooler operation of the engine, particularly at heavy loads.

Demonstration Magnetic Amplifier for Educational and Laboratory Use.—With high performance self saturating magnetic amplifiers assuming a role of increasing importance in control applications, Vickers Electric Division (St. Louis) of Vickers Incorporated has designed a demonstration unit for educational purposes in schools and industry.

A static device with life and maintenance characteristics comparable to those of transformer and similar devices, the magnetic amplifier has been substituted for vacuum and gas discharge tubes—and has many unique characteristics which permit new results in control engineering. The new Vickers demonstration unit has been developed to enable students, research personnel and others to gain wider knowledge of these basic characteristics.

The new Vickers demonstration magnetic amplifier is arranged so that all basic single-phase self-saturating circuits may be studied. By arrangement of the external connections either d-c. or a-c. output is available and either d-c. or a-c. control power may be used. As the characteristics of amplifiers connected for d-c. and a-c. output and control differ, it is important that all of these circuits and their combinations be studied.

Aside from the basic purpose to show the principle of the high-performance self-saturating magnetic amplifier, the new demonstration unit may be used in operating control circuits, such as voltage regulators for small a-c. and d-c. generators, speed control for small d-c. motors, controlled rectifier power supply, and closed-loop positioning systems commonly employed in servo-mechanism work. Reference material and bulletins included with each unit give the wiring diagrams of several control circuits in which this magnetic amplifier may be used, as well as a series of laboratory experiments arranged to help the student determine magnetic amplifier static characteristics, internal impedance and optimum load, dynamic characteristics and types of control current. Technical data on the application, use and performance characteristics of the Vickers Standard Magnetic Amplifiers is also included with each demonstration unit as further aid to the student in supplementing his knowledge of magnetic amplifiers.

All colleges, universities, laboratories and other groups who are interested in the new Vickers Demonstration Magnetic Amplifier are invited to write for complete details to the Vickers Electric Division, 1815 Locust Street, St. Louis 3, Missouri.

X-Y Recorder.—To provide information about a new high-speed curve-plotting recorder valuable in research and routine investigations for drawing the relationship between two functions, Leeds & Northrup Company has issued an illustrated 4-page folder describing its new Speedomax X-Y Recorder.

The folder emphasizes the breadth of possibilities in plotting results, under test, of many types of investigations such as hysteresis loops of iron samples, temperature vs strain in structural materials, vacuum tube characteristics, computer output—in fact, any pair of variables which can be brought to the instrument in the form of d-c. voltages.

The operating principles of the instrument, which employs two electrical balancing systems—one for pen motion, the other for chart travel—are described with the help of a simplified schematic diagram.

Specifications, including ranges, response speeds and construction details, are also given. Photographs illustrate the compactness and ease of operation.

A copy of Folder EM9–420(1), describing the Speedomax X-Y Recorder, will be sent on request. Address Leeds & Northrup Company, 4934 Stenton Avenue, Philadelphia, 44, Pennsylvania.

Flame Detector (*Electrical World*, Vol. 133, No. 7).—Instantaneous response to flame conditions to prevent a discharge of unburned fuel to combustion chambers, and shut off the fuel supply in event of failure, is claimed for a flame detector used with oil or gas fired burners. Known as a Protectoglo, it operates on flame rectification principles. The system applies alternating current to either a flame-electrode immersed in the flame, or to a photocell sighted at the flame, to rectify the current.

The detector can be interlocked with any needed combinations of control devices and accessory equipment such as excess-pressure and low-water safety cutoff or temperature controls. It is applicable to single or multiple burner installations. For further information, write the Minneapolis-Honeywell Regulator Co., 2953 Fourth Ave., S., Minneapolis 8, Minn.